Sample of Typical Final Examination Problems

Math 130 Precalculus for the May 24, 2013 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. Find a third point on the line containing the points $A(-3, 8)$ and $B(7, 2)$ that's just as far from B as A is.

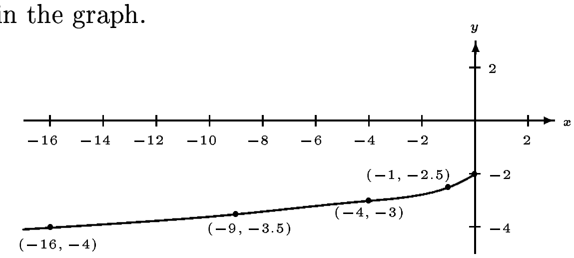
   Explain what characterizes any other point on that line, and point out which formula would be best to provide the coordinates of the unique point on that line which is located just as far from B as A is.

2. Write the standard form of the equation of a circle with endpoints of a diameter $(0, 0)$ and $(4, -6)$. State the center and the radius, and sketch the graph of the circle, labelling all intercepts with their coordinates.

3. Find an equation of the line passing through the points $\left(2, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{5}{4}\right)$. Sketch the line and label both intercepts with their coordinates.

4. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\dfrac{f(x+h) - f(x)}{h}$ and simplify the result.

5. Identify the parent function and the transformation shown in the graph. Then write an equation for the function shown in the graph.



6. For $f(x) = \sqrt{x+4}$ and $g(x) = x^2$, find $f \circ g$ and $g \circ f$.

   Find the domain of each function and the domain of each composite function.

7. Determine whether $h(x) = -5x + 3$ has an inverse function. If it does, then find the inverse function.

8. Write the quadratic fuction $f(x) = \frac{1}{4}x^2 - 2x - 12$ in standard form and sketch its graph.

   Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.

9. Find two positive real numbers whose product is a maximum, under the condition that the sum of the first number and three times the second number is 42.

10. Graph $y = 5^x$.

    For each of these equations, state the transformation of $y = 5^x$ that yields the new equation, and then graph the transformed equation.

    (a) $y = 5^x - 2$.

    (b) $y = 5^{x-2}$.

    (c) $y = \frac{1}{25}\left(5^x\right)$.

    Are any of these three new equations equivalent? If so, prove it algebraically and show that their graphs have the same points.

11. Which of these are equivalent? $y = \left(\frac{1}{9}\right)^{x-2}$; $y = 81\,(9^{-x})$; $y = 81\,(3^{-2x})$; $y = \frac{81}{9^x}$; $y = \left(\frac{9}{3^x}\right)^2$

12. Let $w = \log_2 a$.
   Find an expression in terms of w for:
   (a) $\log_2 a^4$
   (b) $\log_2 (4a^2)$
   (c) $\log_2 (4a)^2$
   (d) $\log_2 (\sqrt[4]{a})$
   (e) $[\log_2 4a]^2$
   (f) $\sqrt{\log_2 a^2}$

13. (a) Simplify to a rational number:
   $$\log_2 \left(16\sqrt[3]{1/4}\right).$$
   (b) Simplify to a rational number or to an exact decimal:
   $$\left[1 + 3\log_2\left(\sqrt[4]{2}\right)\right]^2.$$

14. Simplify: $16^{-2+3\log_{16} 5}$. Give the answer as an exact rational number.

15. In each of parts (a) through (h), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 1.098$, $\log_b 3 \approx 1.740$, and $\log_b 5 \approx 2.5495$.
   (a) $\log_b 6$
   (b) $\log_b \frac{3}{5}$
   (c) $\log_b 125$
   (d) $\log_b \sqrt{3}$
   (e) $\log_b 20$
   (f) $\log_b (4b)^{-2}$
   (g) $\log_b (5b^2)$
   (h) $\log_b \sqrt[3]{2b}$

16. Is knowing that $\log_b 7 = 2.472$ enough to find $\log_b 10$? If so, find $\log_b 10$.

17. Solve algebraically:
   (a) $\log x + \log(x - 15) = 2$.
   (b) $\log x - \log(x - 15) = 2$.
   (c) $\log_8 (x + 1) - \log_8 (x - 3) = \frac{1}{3}$.
   (d) $\log 24x - \log(1 + \sqrt{x}) = 2$.

18. Decide if each statement is true or false. Then justify your answer by writing an equation.
   (a) Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.
   (b) Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.

19. The number of bacteria present in a culture is increasing according to the law of exponential growth. After 3 hours there are 100 bacteria and after 7 hours there are 400 bacteria. How many bacteria will there be after 8 hours?

20. (a) Rewrite in radian measure as a multiple of π: (i) 130° (ii) -60°

    (b) Rewrite in degree measure: (i) $\frac{3\pi}{2}$ (ii) $\frac{5\pi}{4}$ (Do not use a calculator.)

    (c) Find the length of the arc on a circle of radius 3 meters intercepted by a central angle of 150°.

21. A carousel with a 50-foot diameter makes 4 revolutions per minute.

    (a) Find the angular speed of the carousel in radians per minute.

    (b) Find the linear speed (in feet per minute) of the platform rim of the carousel.

    (c) If an individual sat at the rim and rode for an hour, how many miles (rounded to two decimal places) would he have travelled?

22. A right triangle has an acute angle θ with $\sec\theta = \frac{3}{2}$. Find the exact values of the other five trigonometric functions of θ, in fractional form.

    Hint. First sketch a right triangle corresponding to that secant. Next use the Pythagorean Theorem to determine the third side. Then find the other five trigonometric functions of θ.

23. Simplfiy each trigonometric expression in (a)–(f) so that the answer is in terms of at most one of the six trigonmetric functions.

    (a) $\sin x \cot x$ (b) $\tan^2 x - \sec^2 x$ (c) $\csc^4 x - \cot^4 x$ (d) $\frac{1}{\cos^2 x \sin^2 x} - \csc^2 x$ (e) $\frac{\cos^2 x - 1}{\tan^2 x}$ (f) $\frac{\sin^2[(\pi/2) - x]}{\cot^2 x}$

24. Find all solutions of $\sin x - 2 = \cos x - 2$ in the interval $[0, 2\pi)$. (Get the algebraically-assisted, exact result.)

25. Find all solutions of $\sec x + \tan x = 2$ in the interval $[0, 2\pi)$.

    Answers may be presented as exact values in terms of inverse trig functions, or in degree or radian form rounded to four decimal places.

26. Find the exact (algebraically-assisted) solutions of the equation in the interval $[0, 2\pi)$. Then graph the double-angle function and the other trig function on the same axes and estimate their points of intersection. Show that these estimates are compatible with the exact solutions.

    (a) $\cos 2x - \cos x = 0$.

    (b) $\sin 2\alpha - \cos\alpha = 0$.

27. Find the exact values of the sine, cosine, and tangent of $\frac{11\pi}{12}$. (No credit for approximate decimal answers.)

    Some people will break the angle down as $\frac{3\pi}{4} + \frac{\pi}{6}$, while others will break the angle down as $\frac{2\pi}{3} + \frac{\pi}{4}$. Won't this make a big difference in the answers?

28. Given that $\cos u = -4/5$ with $\pi/2 < u < \pi$, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

29. What can you say about the trig functions of double the angle 323.13° as compared with the trig functions of double the angle 143.13°?

30. Use the Law of Cosines to solve the triangle with sides of lengths 6 and 7, and an included angle of 120° between them. (Round angles to three decimal places.)

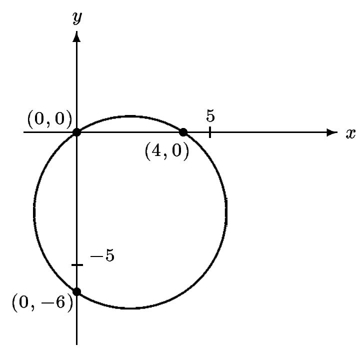
Answers to the Sample of Typical Final Examination Problems

Math 130 Precalculus for the May 24, 2013 Final Exam

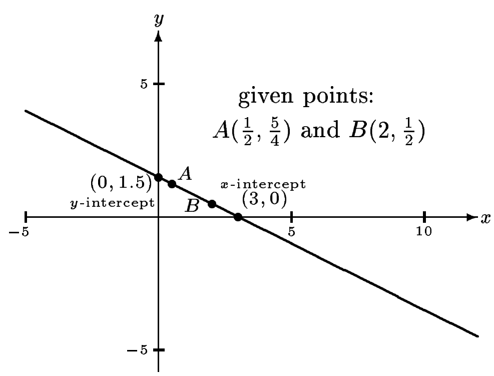
1. The point is $(17, -4)$. A line segment connecting any other point on that line to either A or B has the same slope, $-3/5$, as does AB.

   But in this case if B is equally as far away from A and C and on the same line, it must be the midpoint of the line segment AC. So set $B(7, 2)$ to the result of finding the midpoint between $(-3, 8)$ and (x, y). Then solve the equations $7 = \dfrac{-3 + x}{2}$ and $2 = \dfrac{8 + y}{2}$ to get $(17, -4)$.

2. $(x-2)^2 + (y+3)^2 = 13$. The center is at $(2, -3)$ and the radius is $\sqrt{13}$.



3. $y = -\frac{1}{2}x + \frac{3}{2}$ or $y - \frac{1}{2} = -\frac{1}{2}(x - 2)$ or $x + 2y = 3$.



4. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{(2x^2 + 4xh + 2h^2 + 3x + 3h - 1) - (2x^2 + 3x - 1)}{h} = \dfrac{4xh + 2h^2 + 3h}{h} = 4x + 2h + 3,\ h \neq 0.$

5. Reflection in the x-axis, vertical shrink by a factor of one-half, and vertical shift two units down of the parent function $y = \sqrt{x}$. The function shown in the graph has equation $y = -\frac{1}{2}\sqrt{-x} - 2$.

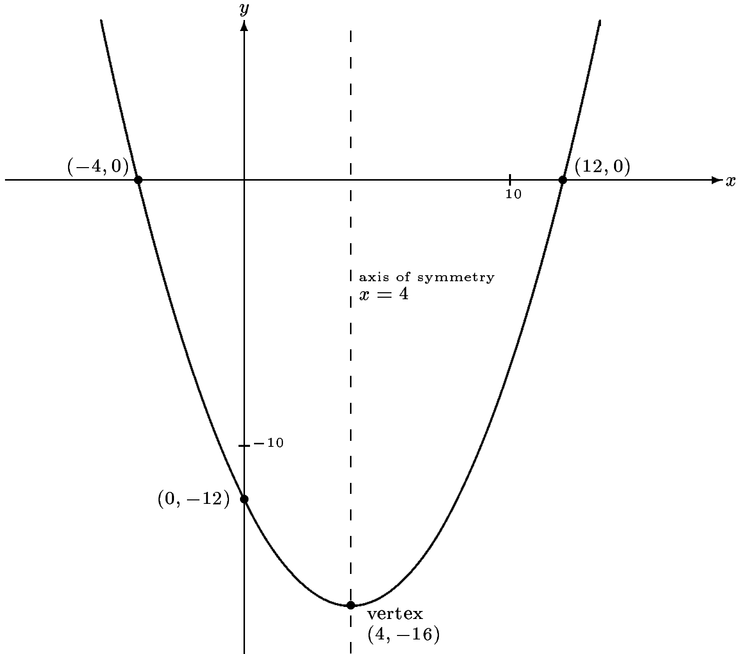
6. (a) $\sqrt{x^2 + 4}$ (b) $x + 4$

   Domains of f and $g \circ f$: all real numbers x such that $x \geq -4$

   Domains of g and $f \circ g$: all real numbers

7. Yes; $h^{-1}(x) = \dfrac{3 - x}{5}$.

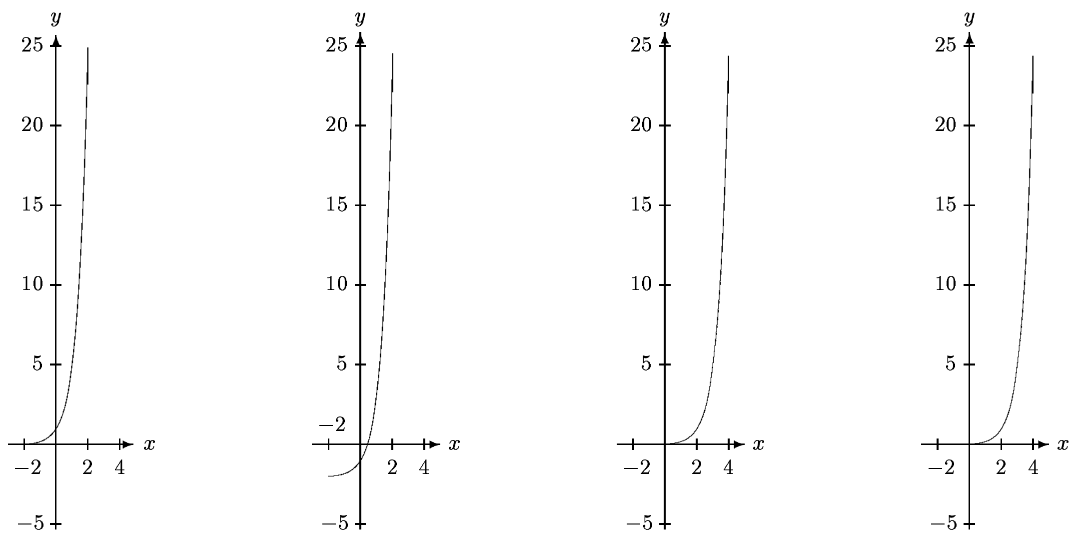
8. $f(x) = \frac{1}{4}(x-4)^2 - 16.$



9. 21, 7

10. Original equation. (a) Verical shift two units down. (b) Horizontal shift two units to the right. (c) Vertical shrink of one twenty-fifth.

This is $f(x)$. This is $f(x)-2$. This is $f(x-2)$. It's $cf(x)$: $c = \frac{1}{25}$ and $0 < c < 1$.



By inspection, graphs (b) and (c) look the same. Algebraically: $5^{x-2} = 5^x/5^2 = 5^x/25 = \left(\frac{1}{25}\right)5^x$.

Take three points on the original graph and transform them: The original graph, $y = 5^x$, has points $(-2, 0.04)$, $(0, 1)$, and $(2, 25)$. Graph (a) has points $(-2, -1.96)$, $(0, -1)$, and $(2, 23)$. Graph (b) has points $(0, 0.04)$, $(2, 1)$ and $(4, 25)$. Graph (c) has points $(-2, 0.0016)$, $(0, 0.04)$, and $(2, 1)$.

Since both the original equation and equation (a) have a point on the graph with $x = 0$ that differs from the others, they cannot be equivalent to any of these equations. But equations (b) and (c), so far, seem to have the same graph. The definition of equivalent equations is that they have the same solution set and the same graph. So, just by looking at the points alone without the algebraic equivalence, it appears that (b) and (c) are equivalent equations.

11. all of them

12. (a) $4w$ (b) $2w+2$ (c) $2w+4$ (d) $w/4$ (e) w^2+4w+4 (f) $\sqrt{2w}$

13. (a) 10/3 (b) 49/16 or 3.0625

14. 125/256

15. (a) 2.838 (b) −0.8095 (c) 7.6485 (d) 0.58 (e) 4.7455 (f) −6.392 (g) 4.5495 (h) 0.69933

16. Yes. it can be determined that $\log_b 10 \approx 2.9251$

17. (a) $x = 20$ (b) $x = 500/33$ or $x = 15\frac{5}{33}$ (c) $x = 7$ (d) $x = 25$

18. (a) True.
    $\log_a uv = \log_a u + \log_a v$
    This is the first prpoerty of logarithms.

    (b) False.
    $(\log_a u) \div (\log_a v) = \log_a(u - v)$
    $2 = \log 100 \div \log 10 \neq \log(100 - 10) = \log 90 \approx 1.95$

19. About 566 bacteria

20. (a) (i) $\frac{13\pi}{18}$ (ii) $-\frac{\pi}{3}$

(b) (i) 270° (ii) 225°

(c) 2.5π meters ≈ 7.85398 meters

21. (a) 8π rad/min ≈ 25.1327 rad/min (b) 628.31853 ft/min (c) 7.14 miles

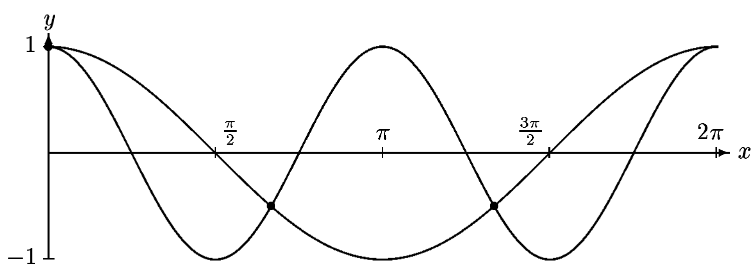
22. $\sin\theta = \frac{\sqrt{5}}{3}$ $\cos\theta = \frac{2}{3}$ $\tan\theta = \frac{\sqrt{5}}{2}$ $\csc\theta = \frac{3\sqrt{5}}{5}$ $\cot\theta = \frac{2\sqrt{5}}{5}$

23. (a) $\cos x$ (b) -1 (c) $2\cot^2 x + 1$ (d) $\sec^2 x$ (e) $-\cos^2 x$ (f) $\sin^2 x$

24. $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

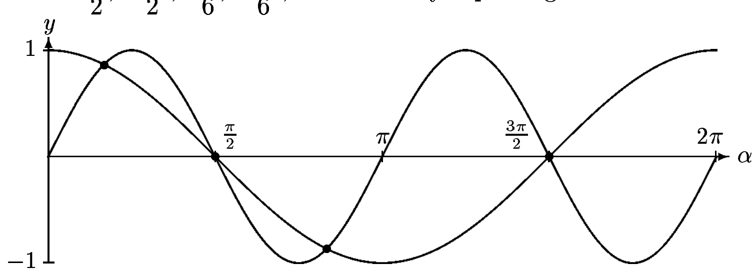
25. $\sin^{-1} 0.6$ or 36.8699° or 0.6435 radians

26. (a) $0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}$; obtained by replacing $\cos 2x$ with $2\cos^2 x - 1$ and solving for x.



The two graphs indeed look like they intersect when $x = 0$, $x = \frac{2}{3}\pi$, and $x = 1\frac{1}{3}\pi$. The apparent fourth point, at $x = 2\pi$, is not in the interval given, as it is just a one-period shift on the cosine graph of the point when $x = 0$.

(b) $\frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{\pi}{6},\ \frac{5\pi}{6}$; obtained by replacing $\sin 2\alpha$ with $2\sin\alpha\cos\alpha$ and solving for α.



The two graphs obviously intersect when $\alpha = \frac{\pi}{2}$ and when $\alpha = \frac{3\pi}{2}$. The other two points are clearly between 0 and $\frac{\pi}{4}$, and between $\frac{3\pi}{4}$ and π, so the actual values are reaonable guesses.

27. $\sin\frac{11\pi}{12} = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3}-1\right)$; $\cos\frac{11\pi}{12} = -\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$; $\tan\frac{11\pi}{12} = -2+\sqrt{3}$
It doesn't matter; either method you choose to break up the angle yields the same result.

28. $\sin 2u = -\frac{24}{25}$, $\cos 2u = \frac{7}{25}$, $\tan 2u = -\frac{24}{7}$

29. Each trig function will take on the same value in both cases, because the double angles will differ by 2π.

30. The third side is $\sqrt{127}$ (about 11.2694) long, the angle opposite the side with length 6 is 27.4571°, and the angle opposite the side with length 7 is 32.5429°. Note that these 3 angles add to 180.0000°.

For December 14, 2012 Math 130 Final

Here are lists of problems from the Repository that might be most relevant.

Setup of exam: 3 long questions on lines, quadratic completing the square, and phase shift graphing of sine or cosine. (These are 10 points each.) Look at problems 1, 2, 4, 14, 79, and 80.

14 short problems worth 5 points each.

2 bonus problems worth 6 points each. They are modelled after problems 5 and 34.

Best to concentrate on these problems out of the 109 problems in the repository when studying for the final exam: 1, 2, 4, 5, 6, 14, 17, 18, 24, 26, 28e, 30, 34, 39, 52, 64, 79, 80, 82, 84, 96, 97, 98, 99, 108.

When getting ready for Math 140 or Math 145, it might not be a bad idea to look at as many of the entire 109 as you can.

Repository of Potential Final Examination Questions

Math 130 Precalculus December 14, 2012

On actual final: no books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. If $y = f(x)$ is a linear function with $f(-2) = 7$ and $f(3) = -3$, find a formula for f.

2. Show that the points $(-8, -65)$, $(1, 52)$, and $(3, 77)$ do not lie on a straight line.

3. Are these statements true or false? Give an explantion for your answer.

   (a) A function is a rule that takes certain inputs and assigns to each input exactly one output value.

   (b) The line $3x + 5y = 7$ has slope $3/5$.

   (c) The line $4x + 3y = 52$ intersects the x-axis at $x = 13$.

   (d) If a line is increasing, any line perpendicular to it must be decreasing.

4. Let A be the point with coordinates $(13.5, 4.5)$ and B be the point with coordinates $(22.5, 44.5)$.

   (a) For the line segment AB: find its length, the coordinates of its midpoint, and its slope.

      i. Find an equation of the perpendicular bisector of AB. Then graph it, and plot with coordinates both intercepts and the point on it where $x = 58$.

   (b) Find an equation of the circle of which AB is a diameter. Then roughly graph it, plotting and labelling with coordinates any seven points on the circle.

5. (a) Find the distance from the point $(3, 4)$ to the line containing the points $(1, 5)$ and $(-2, 2)$.

   One method might be to draw a right triangle with vertical and horizontal legs that meet at the point $(3, 4)$ so that its hypotneuse is on the line containing the points $(1, 5)$ and $(-2, 2)$. Find the equation of the line; then use it to find the points on the line where $x = 3$ and $y = 4$. They are the endpoints of the hypotenuse. Since the area of this triangle is readily found, the area can be set equal to half of the product of the length of the hypotenuse (which can be directly derived from the coordinates of the endpoints) and the distance that is to be found. Then simply solve for the distance. This assumes that the distance is measured along a line perpendicular to the line containg the two given points.

   There is a fancy variation whereby areas are not needed. The distance d to be found is to the horizontal leg as the vertical leg is to the length of the hypotenuse because the larger right triangle is similar to both of the smaller right triangles, and either smaller triangle may be chosen to get the proportion. Again this assumes that the distance is measured along a line perpendicular to the line containing the two given points.

   A second method is to find the equation of tbe line and then find the equation of a line through the point $(3, 4)$ that is perpendicular to it and derive the point of intersection. The distance of a point to a line is always the length of the perpendicular from the point to the line. Use the distance formula on the two points.

   A third method would be to take the distance from an arbitrary point on the line, calling the point (a, b), and minimize the square of that distance. From the equation of the line, b can be replaced by $ma + b$. The square of the distance is a quadratic function of a and the a that makes is smallest can be found. Finally plug that a into the quadratic equation and take the square root of the result.

   (b) Use the information from part (a) to find the area of the triangle whose vertices are $(3, 4)$, $(1, 5)$, and $(-2, 2)$.

6. Find the center, radius, diameter, and circumference of the circle in the xy-plane described by the equation

$$x^2 + 5x + y^2 - 6y = -3.$$

   Find the coordinates of the lowest point and of all intercepts. Does this circle have any points on its graph that lie in the fourth quadrant?

7. Graph over the real numbers $y = \sqrt{-\sqrt{-x}}$.

   State the domain, range, and the quadrants with points on the graph. Then list all points on the graph.

8. Simplify if possible; then graph $y = -|-x|$. Decide whether the graph opens up or down.

   Plot and label with coordinates the points, if any, where $x = -8$, $x = 0$, $x = 2$, $y = -8$, $y = 0$, and $y = 2$.

9. (a) Graph on the same axes the functions $y = \sqrt{2x}$ and $y = |x - 4|$, sketching at least as far as $x = 12$. Estimate from the graph the solutions to the equation $\sqrt{2x} = |x - 4|$.
   (b) Solve $\sqrt{2x} = |x - 4|$ algebraically and show that the solutions are the same.

10. For the function f: find the domain, range, intercepts, and the asymptotes. Then draw a rough graph and label the intercepts with coordinates and the asymptotes with equations.

$$f(x) = 1/(x + 1) + 3.$$

Now find $f(-2)$, $f(2)$, $f^{-1}(-2)$, and $f^{-1}(2)$. Then plot the corresponding points on the graph of f, labelling them with coordinates.

11. Consider the fucntion defined by the formula

$$f(x) = 1/(x + 1) + 3.$$

Is this function one-to-one? Is its inverse a function?

If you answered yes to the above, find a formula for $f^{-1}(x)$ and use that formula to find $f^{-1}(7)$.

State the domain of f and the domain of the inverse of f.

12. Find $-f(-x)$ for the function $f(x) = x^3 + 7$.

13. For the function $g(x) = x^2 - 7x + 4$, find

$$\frac{g(a) - g(-a)}{2}.$$

14. Complete the square of $f(x) = -3x^2 + 5x - 1$, getting it into standard form.
    Standard form is often written as $a(x - h)^2 + k$ or $k(x + t)^2 + r$.

Find the value of x where $f(x)$ attains its minimum value or its maximum value.

Sketch the graph of f on the interval $\left[-1, 2\tfrac{2}{3}\right]$ (that is for $-1 \leq x \leq 2\tfrac{2}{3}$).

Find the vertex of the graph of f and the equation of the line of symmetry.

Is f an odd function, an even function, neither, or both?

15. An object is thrown up from a balcony so that its height above ground in meters at time t seconds when $t \geq 0$ and $y \geq 0$ is given by the equation

$$y = -5t^2 + 18t + 7.$$

   (a) Complete the square and graph, labelling with coordinates the vertex, and all t- and y-intercepts (even those where $t < 0$). Then graph the line of symmetry and label it with its equation.
   (b) How high is it initially? Does it ever get that high again. If so, when?
   (c) When is the object the highest? How far above the ground is it at that moment? When, if ever, does it reach that height again?
   (d) When, if ever, does it attain a height of 20 meters above the ground?
   (e) When does the object hit the ground?

16. Let $f(t) = -5t^2 + 18t + 7$.
   (a) Find and simplify an expression for $\dfrac{f(t + h) - f(t)}{h}$, assuming $h \neq 0$. (Answer is in terms of t and h.)
   (b) Let $g(x) = \dfrac{1}{x + 4}$.

   Find and simplify and expression for $\dfrac{g(2 + h) - g(2)}{h}$, assuming $h \neq 0$.

17. Assuming that $g(x) = \dfrac{1}{1+3x}$ and that $h \neq 0$, evaluate and simplify the expression

$$\frac{g(x+h) - g(x)}{h}.$$

18. Find two numbers whose sum equals 120 and whose product equals 3519. Do not use a calculator.

19. A right triangle is to be drawn so that the sum of the legs is 120 cm. What's the largest possible area?

20. A city has $4N$ yards of fencing to create an enclosed area in a public park.

    (a) Find the largest possible area if the shape is to be rectangular. The answer will be in terms of N.

    (b) Find the largest possible area if the shape is to be circular. Give answer in terms of N and π.

    (c) Which gives the larger area? By what percent does its area exceed the other?

21. Assume that $0 \leq p \leq 1$. Determine the largest value that $\sqrt{p(1-p)}$ can take on.

22. Let $f(x) = \dfrac{1}{x}$ and $g(x) = \sqrt{x}$.

    True or false?

    (a) The product function, the composition of f with g, and the composition of g with f are all the same.

    (b) The inverse of f is the same as f.

23. Let $f(x) = \dfrac{1}{x+1}$.

    (a) Find $f\left(\dfrac{1}{x}\right)$, $\dfrac{1}{f(x)}$, and $f^{-1}(x)$.

    (b) Find and simplify the product: $\left[f^{-1}(x)\right] \cdot \left[f\left(\dfrac{1}{x}\right)\right] \cdot \left[\dfrac{1}{f(x)}\right]$.

    (c) Find $f^{-1}(f(x))$.

24. Suppose $h(t) = \dfrac{2-3t}{4+5t}$.

    (a) Show that h is a one-to-one function.

    (b) Find a formula for h^{-1}.

25. Let $f(x) = \dfrac{3}{x-3} - 3$, $g(x) = x^4 - 2$, and $h(x) = x + 3$.

    (a) If g has an inverse function, find a formula for g^{-1}, the inverse of g.

    If f has an inverse function, find a formula for f^{-1}, the inverse of f. Is f its own inverse?

    Find $f(2)$, $f^{-1}(-6)$, $f(0)$, $f^{-1}(-4)$, $f(2\sqrt{3})$, and $f^{-1}(2\sqrt{3})$. Then explain how these results validate your formula for f^{-1} or support your conclusion that f does not have an inverse function.

    (b) Find formulas for $(f \circ h)(x)$ and for $(h \circ f)(x)$. Find a formula for $\left(f^{-1} \circ f\right)(x)$.

26. For the functions $f(x) = \dfrac{6x+3}{x^2 - 2x + 1}$ and $g(x) = \sqrt{x+1}$, find a formula for and the domain of:

    (a) $f \circ g$.

    (b) $g \circ f$.

    Simplify your results as much as possible.

27. Let $f(x) = 3x^2$, $g(x) = 9x - 2$, $m(x) = 4x$, and $r(x) = \sqrt{3x}$.
    Simplify the composite functions: (a) $f(r(x))$, (b) $r(f(x))$, (c) $g(m(f(x)))$.

28. Let $f(x) = x^2 + x$ and $g(x) = \dfrac{x}{1-x}$.

    Find formulas for: (a) $\big(f(x)\big)^2$, (b) $f(x^2)$, (c) $f^2(x)$, (d) $g^{-1}(x)$, (e) $\big(g(x)\big)^{-1}$, (e) $f(g(x))$.

29. Let $g(x) = x^2 - 4$.

    State the domain and range.

    Evaluate and simplify: (a) $g^2(x)$, (b) $\big(g(x)\big)^2$, (c) $g(x^2)$, (d) $\sqrt{g(x)}$, (e) $g(\sqrt{x})$, (f) $g(x+1)+1$.

30. Find the domain and range of the function f defined by $f(x) = \log_4 x$.

    Graph it on the interval $[0.625, 32]$, plotting and labelling with coordinates all points with integer y. Also plot the endpoints and label them with their coordinates.

31. Find the domain of the following funtions.

    (a) $h(x) = \ln\left(x^2\right)$
    (b) $g(x) = (\ln x)^2$
    (c) $f(x) = \ln(\ln x)$
    (d) $k(x) = \ln(x-3)$

32. True or false?

    (a) $\log AB = \log A + \log B$.
    (b) $\dfrac{\log A}{\log B} = \log A - B$.
    (c) $\log A \log B = \log A + \log B$.
    (d) $p \cdot \log A = \log A^p$.
    (e) $\log \sqrt{x} = \frac{1}{2} \log x$.
    (f) $\sqrt{\log x} = \log\left(x^{1/2}\right)$.

33. Suppose that $a = \log 2$.

    Find possible formulas for the following expressions in terms of a. Your answers should not involve logs.

    (a) $\log 0.4$
    (b) $\log 0.25$
    (c) $\log 40$
    (d) $\log \sqrt{5}$
    (e) $\log\left(10^{5a}\right)$
    (f) $10^{a/3}$ (this answer will not be in terms of a)
    (g) $\log\left(10^{5a} - 16\right)$
    (h) $\log\left(10^{\sqrt{a}}\right)$

34. Assume that $\log_4 a = 1.4$ and $\log_4 b = 3.3$. Evaluate each of the following quantities.

    (a) $\log_4(2ab)$.
    (b) $\log_4 \frac{b}{4a}$
    (c) $\log_4 \sqrt{a}$
    (d) $\log_4 \frac{1}{a^3}$
    (e) $\log_8 b^{20}$
    (f) $\log_a 4$

35. (a) Simplify i. $\dfrac{\log x}{4} + \log\left(\dfrac{x}{4}\right) - \log\left[\left(\dfrac{x}{10}\right)^2\right] - \log 25.$ ii. $4^{\log_2 6} - 4^{\log_2 3}.$ iii. $4^{\log_2 6 - \log_2 3}.$

iv. Simplify to an exact value $2^{\left(\frac{1}{\log 2}\right)}.$ v. Simplify (exact fraction, no decimals) $9^{4 - \log_3 2}.$

(b) Solve. Then check your answer(s) in the original equation.

i. $\log x + \log(25 - x) = 2.$

ii. $\log_2(1 - 3x) + \log_2 2x = -3.$

iii. $\dfrac{\log(x^3 + x^2 - 260x)}{\log x} = 3.$

iv. $\dfrac{\log_6(5x - 21)}{\log_6(x - 3)} = 2.$

36. Solve, without a calculator, the equation $250^p = 25$, using 0.3 as an approximation to $\log 2$.
The answer will be an exact fraction. It turns out that this result is accurate to three decimal places.

37. Simplify fully: $\ln(A + B) - \ln\left(A^{-1} + B^{-1}\right) - \ln B$. Assume that both A and B are positive.

38. Find a number b such that $\log_b 9 = -2.$

39. Find a number such that $\log_4(3x + 1) = -2.$

40. Solve for x: $\log_4(1 - 3x) + \log_4 x = -2.$
Check each answer to show that it satisfies the original equation.

41. Solve for x: $\log_4(3 - 4x) + \log_4(2x - 1) = -3/2.$
Check all answers to show that they satisfy the original equation.

42. Solve for x: $\log_3\left[(2x - 1)^2 \cdot x^2\right] = 2.$
To establish validity, check each answer in the original equation.

43. Solve for x: $\log\left[(3x + 1)(x + 2)\right] = 2.$
To establish validity, check each answer in the original equation.

44. Solve for x: $\log x - \log(x - 1) = 1/2.$
Find the answer(s) in exact radical form. Then approximate to six decimal places on your calculator and substitute into the original equation for verification.

45. Solve for x:

(a) $\log(4.5 - 2x) \cdot \log x = 0.$

(b) $\log(4.5 - 2x) + \log x = 0.$

46. Solve for x: $\log(4x - 2) \cdot \log(5x - 2) = 0.$

47. Solve for x: $\log(2x - 1) \cdot \log(2x + 1) = 0.$

48. Solve for x: $\log\left[x^x (x - 4.5)^x\right] = 2x.$

49. Let $Q(t) = 8(0.87)^t$ give the level of a pollutant (in tons) remaining in a lake after t months.
What is the monthly rate of decrease of the pollutant? The annual rate? The daily rate?

50. A population grows exponentially according to the formula $P = 25(1.075)^t$.

(a) What is the initial value of P (when $t = 0$)?

(b) What is the percent growth rate?

(c) What is the growth factor for one unit of time?

(d) Use logs to find the exact value of t when $P = 100$.

51. A toxic asset balance grows exponentially. Initially it was \$1411.02; at time $t = 3$ days it was \$1411.37.
    (a) Find an expression which gives the balance as a function of t in days.
    (b) Find the toxic asset balance at time $t = 90$ days and at the end of four years (at time $t = 1461$ days).
    (c) When, if ever, is the balance \$2,000?
    (d) Find the doubling time in days. Based on that, at which time in years will the balance be \$11,288.16?

52. Suppose f is a function with exponential growth such that

$$f(1) = 3 \text{ and } f(2) = 4.$$

Evaluate $f(0)$ and $f(3)$; and find the doubling time of f.

53. On July 10, 2012, the population of the United States was about 313.9 million and increasing at a rate of 0.91% per year. In what year is the population projected to reach 400 million?

54. A colony of bacteria is growing exponentially. At the end of 3 hours there are 1000 bacteria. At the end of 5 hours there are 4000.
    (a) Write a formula for the population of bacteria at time t, in hours.
    (b) By what percent does the number of bacteria increase each hour?

55. Suppose a colony of bacteria starts with 200 cells and triples in size every four hours.
    (a) Find a function that models the population growth of this colony of bacteria.
    (b) Approximately how many cells will be in the colony after six hours?

56. Forty percent of a radioactive substance decays in five years, By what percent does the substance decay each year?

57. If 17% of a radiactive substance decays in 5 hours, what is the half-life of the substance?

58. Find the annual growth rates of a quantity which:
    (a) Doubles in size every 7 years.
    (b) Triples in size every 11 years.
    (c) Grows by 3% per month.
    (d) Grows by 18% every 5 months.

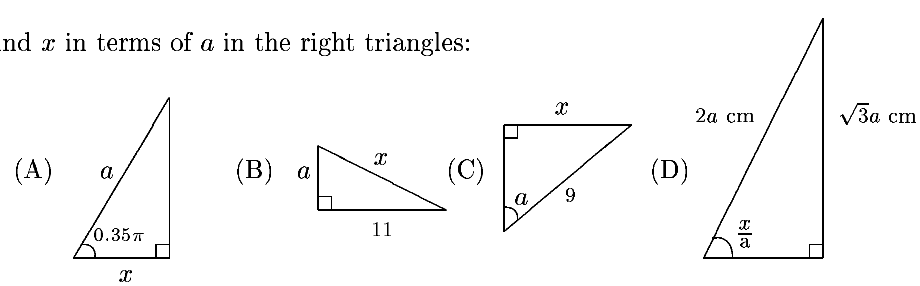
59. The population of bacteria m doubles every 24 hours.
    The population of bacteria q grows by 3% per hour.
    Which has the larger population in the long run?

60. The temperature T in degrees Fahrenheit of a potato is given by the equation

$$T(t) = 425 - 355e^{-.06t},$$

where t is the time in minutes after it is placed in the oven.
    (a) Graph $T(t)$ for $t \geq 0$, labelling with coordinates the points when $t = 0$, $t = 10$, $t = 60$, and $t = 200$, and graphing the asymptote and labelling it with its equation. When $t = 60$, how long has the potato been in the oven?
    (b) When, if ever is the temperature of that potato: 350 degrees Fahrenheit? 450 degrees Fahrenheit?

61. Find x in terms of a in the right triangles:



62. A ladder 3 meters long leans against a house, making an angle α with the ground. How far is the base of the ladder from the base of the wall, in terms of α? Include a sketch.

63. Hampton is a small town on a straight stretch of coast line running north and south. A lighthouse is located 3 miles offshore directly east of Hampton. The lighthouse has a revolving searchlight that makes two revoloutions per minute. The angle that the beam makes with the east-west line through Hampton is called ϕ.

    (a) Assume that the beam is pointed in a westerly direction and it is hitting the shore.
    Find the distance from Hampton to the point where the beam strikes the shore, as a fucntion of ϕ. Include a sketch.

    (b) Find, in radians per second, the value of ω, the angular velocity of the searchlight.

64. A right triangle has legs of lengths 5 inches and 12 inches.

    (a) Find the area of the triangle.

    (b) A line segment is drawn perpendicular to the hypotenuse from the vertex of the right angle.
        i. Find the length of that line segment.
        ii. This line segment divides the triangle into two parts. Find the area of each part.

    (c) Redraw the right triangle with legs of 5 inches and 12 inches. Connect the vertex of the right angle and the midpoint of the hypotenuse with a line segment, dividing the right triangle into two parts.
        i. How long is that line segment?
        ii. Find the area of each part.
        iii. Find all three angles of the smaller triangle that has one side of length 5.

65. (a) Convert to radians: $330°$ $\pi°$

    (b) Convert to degrees: $\frac{7}{2}\pi$ 2 90 45 $\frac{5\pi}{\pi}$

66. How far does the tip of the minute hand of a watch move in 1 hr., 27 minutes if the hand is 2 inches long?

67. The second hand of a large clock is 20 inches long.

    (a) Find the angulsr velocity in radians per second and the rate of travel—in inches per ssecond—of the tip of the second hand along the circle it sweeps.

    (b) In 25 seconds, find the angle swept by the second hand and the distance along the circle travelled by the tip of the second hand. Also find the straight-line distance between the position of the tip of the second hand and its former position 25 seconds previous.

68. (a) What is the angle determined by an arc of length 2π meters on a circle of radius 18 meters?

    (b) What is the length of an arc which is cut off by an angle of $225°$ in a circle of radius 4 feet?

    (c) What is the radius of a circle in which an angle of 3 degrees cuts off an arc of 30 cm?

69. How many miles on the surface of the earth correspond to one degree of longitude? (The earth's radius is 3960 miles.)
    Give answer as an exact mutliple of π.

70. An ant starts at the point $(1, 0)$ on the unit circle and walks counterclockwise a distance of 3 units around the circle. Find the x and y coordinates (accurate to 2 decimal places) of the final location of the ant.
    How could these 2-decimal answers have been estimated correctly without a calculator or trig table?

71. Let $\phi = 0.93$ be an angle of the unit circle. Its sine is about 0.8.
    Sketch each of these on the unit circle and label the terminal point with both of its approximate coordinates.
    Do not use a calculator.

    ϕ $\pi + \phi$ $\pi - \phi$ $\pi/2 - \phi$ $2\pi - \phi$ 2ϕ $2(\pi/2 - \phi)$ $\frac{1}{2}\phi$ (leave this one as square roots)

72. Let A and B be the points on the unit circle in standard postion that correspond to $\frac{\pi}{12}$ and $\frac{5\pi}{12}$, respectively. Find

- The slope of the line segment AB.
- The exact distance between A and B. (An approximate decimal will not be accepted.)
- The length of the arc of the circle between A and B.
- The size of the central angle AOB in degrees. (O is the center of the circle.)
- The exact coordinates of the midpoint of the line AB (radical form).
- The exact coordinates of the midpoint of the arc AB (radical form).

73. Find the area of the square formed by connecting the four midpoints of the the four quarter-circle arcs of the unit circle. Connect the points with the four line segments that do not include the center of the circle.

74. In which quadrants do the following statements hold?

(a) $\sin\theta > 0$ and $\cos\theta > 0$
(b) $\tan\theta > 0$
(c) $\tan\theta < 0$
(d) $\sin\theta < 0$ and $\cos\theta > 0$
(e) $\cos\theta < 0$ and $\tan\theta > 0$

75. For the graph of each of the following functions, state the period, amplitude, phase shift, midline, and the coordinates of the y-intercept.

(a) $y = \sin(2t)$
(b) $y = (\sin t) + 2$
(c) $y = 2\sin t$
(d) $y = \sin(t + 2)$

76. If possbile, find one exact zero of each of the following functions.

(a) $y = \cos(t + 2)$
(b) $y = 2\cos t$
(c) $y = \cos(2t)$
(d) $y = \cos t + 2$

77. Graph $f(t) = \cos t$ and $g(t) = \sin(t + \pi/2)$. Explain what you see.

78. (a) The function f is an even periodic function with period 6π.
Suppose that $f(\pi/2) = 9\sqrt{3}/2$. Find $f(11\pi/2)$.
(b) Someone claims that he has a function g which is both odd and periodic with period 1, and that $f(\frac{1}{2}) = 2$.
Is anything wrong?
(c) An odd function has a domain that includes 0. Find $f(0)$.
(d) An even function is defined only for the domain $[-3, 11]$.
Is anything wrong?
(e) Someone claims to have found a function g that is symmetric across the line $y = x$ such that the domain of g is all real numbers except 0 and the range of g is all reals.
Is that possible?
(f) Will a periodic function ever have a domain that has a largest value?

79. State the amplitude, period, phase shift, and horizontal shift. Without a calculator, graph the function on the interval $-\frac{3}{2} \le t \le \frac{1}{2}$.

$$y = 3\sin\left(4\pi t + 6\pi\right).$$

Plot and label with coordinates all intercepts, a point where $x = \frac{1}{24}$, and a point where $y = \frac{3\sqrt{3}}{2}$.

80. Sketch the graph of $5\cos\left(2x - \dfrac{\pi}{3}\right)$ for $-\pi/3 < x < 7\pi/6$.

State the range, the amplitude, the period, the fraction of the period which the graph has been shifted, and the direction of that shift.

Plot and label with coordinates the x- and y-intecepts; all highest and lowest points; the point for which $x = \pi/2$; and one point for which $y = 3\sqrt{2}$.

81. Find its period, phase shift, amplitude, and vertical translation; then graph carefully for $-2\pi/3 \le x \le 2\pi$:

$$y = 4\sin\left(\tfrac{3}{4}x + \tfrac{\pi}{2}\right) - 2.$$

Plot and label with coordinates all intercepts, a point where $x = \frac{\pi}{3}$, and a point where $y = 2 + 2\sqrt{3}$.

82. Suppose $\dfrac{\pi}{2} < \theta < \pi$ and $\cos\theta = -\frac{7}{25}$.

Evaluate:

(a) $\sin\theta$
(b) $\tan\theta$
(c) $\sec 2\theta$
(d) $2\sin\theta\cos\theta$
(e) $\sin 2\theta$
(f) $\cos^2\theta - \sin^2\theta$
(g) $\sin\left(\frac{1}{2}\theta\right)$
(h) $\cos\left(\frac{3\pi}{2} - \theta\right)$.

83. Suppose that $\tan x = 2/3$.

Could x possibly be negative?

Evaluate:

(a) $\sin 2x$
(b) $\sin x$
(c) $\cos x$
(d) $\sec^2 x - \tan^2 x$
(e) $\sin(x + \pi)$
(f) $\left|\cos\frac{1}{2}\theta\right|$

84. Suppose that s is in the interval $(0, \pi)$, with $\tan s = \sqrt{5}$.

Find exact expressions for: (a) $\cot s$ (b) $\sec s$ (c) $\sin s$ (d) $\cos s$ (e) $\cos^2 s - \sin^2 s$ (f) $\sec 2s$

85. (a) Find all solutions with $0 \le x \le \pi$ for $\cos x = \tan x$.
(b) Graph $\cos x$ and $\tan x$ on the same axes and indicate on your sketch the points corresponding to the solutions in part (a).

86. Without a calculator, evaluate the following exactly.
(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1}(-1/2)$ (c) $\cos\left(\cos^{-1}(1/2)\right)$ (d) $\cos^{-1}\left(\cos(5\pi/3)\right)$

87. Consider the functions: $f(x) = \sin^{-1}(x)$, $g(x) = \sin\left(x^{-1}\right)$, and $h(x) = (\sin x)^{-1}$.
(a) Evaluate each function for $x = 0.5$. Give an exact answer if possible.
(b) Match each of the following verbal descriptions with one of the three above functions, if possible.
(A) The cosecant. (B) The arcsine. (C) The cosine (D) The negative of the sine.
(E) The reciprocal of the sine. (F) The sine of the reciprocal. (G) Half pi minus the arccosine.
(c) For each of functions f, g, and h, decide if the inverse is a function (whether the original is one to one). If so, find a formula for the inverse function, including its exact domain.

88. Evaluate the following expressions in radians. Give an exact answer if possible.
(a) $\arccos(0.5)$ (b) $\arccos(-1)$ (c) $\arcsin 1$ (d) $\arcsin(0.1)$ (e) $\arccos\left(-\sqrt{3}/2\right)$

89. A tree 50 feet tall casts a shadow 60 feet long. Find the angle of elevation of the sun.

90. Find approximately the acute angle formed by the line $y = -2x + 5$ and the x-axis.

91. The front door to the student union is 20 feet above the ground, and it is reached by a flight of steps. The school wants to build a wheel-chair ramp, with an incline of 15 degrees, from the ground to the door. How much horizontal distance is needed for the ramp?

92. State the domain and range of the following functions and explain what your answers mean in terms of evaluating the functions.
Then draw a rough graph of each labelling any endpoints, asymptotes, and intercepts. Also, for each function describe the concavity for positive x and for negative x. Which of them are increasing and which of them are decreasing?
(a) $f(x) = \sin^{-1}(x)$
(b) $g(x) = \cos^{-1}(x)$
(c) $h(x) = \tan^{-1}(x)$

93. One of the following statements is always true; the other is true for some values of x and not for others. Which is which? Justify your answer with an example.
I. $\arcsin(\sin x) = x$ II. $\sin(\arcsin x) = x$

94. Let a be a number with $0 \le a \le \pi/2$ and let $b = \pi + a$.
What is
(a) $\arccos(\cos a)$? (b) $\arccos(\cos b)$?

95. Simplify the following expressions.
(a) $\dfrac{\cos 2t}{\cos t + \sin t}$
(b) $\dfrac{1}{1 - \sin\theta} + \dfrac{1}{1 + \sin\theta}$
(c) $\dfrac{\cos\phi - 1}{\sin\phi} + \dfrac{\sin\phi}{\cos\phi + 1}$
(d) $\dfrac{1}{\sin t \cos t} - \dfrac{1}{\tan t}$

96. Simplify to a single trig function of a multiple of θ.

$$\cos^2\theta(1+\tan\theta)(1-\tan\theta)$$

97. How are the expressions $(\tan^2 x)(\sin^2 x)$ and $\tan^2 x - \sin^2 x$ related?

98. Decide whether the equation $\tan x = \dfrac{\sin(2x)}{1+\cos(2x)}$ is an identity.

    If it is, prove it algebraically.

    If it is not, find a value of x for which the equation is false.

99. Show that

$$\cos^3\theta + \cos^2\theta\sin\theta + \cos\theta\sin^2\theta + \sin^3\theta = \cos\theta + \sin\theta$$

for every number θ.

100. Use a graph to find all the solutions to the equation

$$12 - 4\cos 3t = 14$$

between 0 and $2\pi/3$ (one period).

How many solutions are there between 0 and 2π?

101. Find approximate decimal values of all solutions with $0 \le t \le \pi/2$ to the equation

$$f(t) = 3 - 5\sin 4t \qquad .$$

(It might be helpful first to draw one period of the graph in the first quadrant starting at $t = 0$.)

102. Simplify to a rational number the expression:

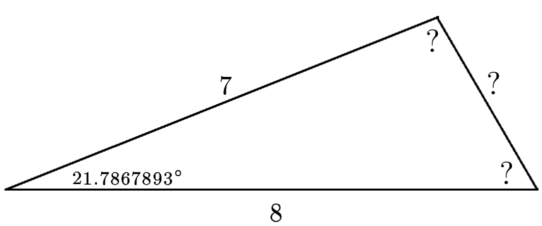
$$\sin\left(2\cos^{-1}\left(\frac{5}{13}\right)\right)$$

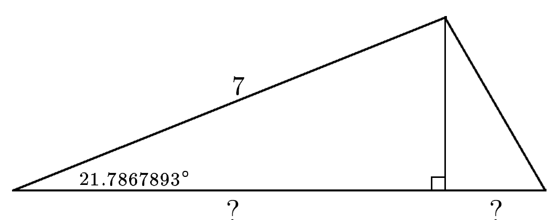
103. (a) Find exactly all solutions to the equation $2\cos^2 x = 3\sin x + 3$ for $0 \le x \le 2\pi$.

     (b) Sketch the graphs of $2\cos^2 x$ and $3\sin x + 3$ on the same axes, indicating on your sketch the points corresponding to the solutions to (a).

     Instead of graphing $2\cos^2 x$ directly, graph an equivalent form in terms of $\cos 2x$. That version will just be a horizontal shrink (change of period) and a vertical translation of the known function $\cos x$.

104. (a) Solve this triangle with sides of lengths 7 and 8 and the included angle equal to 21.7867893 degrees. Round the angles in degrees to 7 decimals. Then show that the three angles add to what's expected.





     (b) Drop a perpendicular from the opposite vertex to the side of length 8. The side of length 8 is now divided into parts of what lengths?

105. Find the third side of a triangle with sides of lengths 4 and $\sqrt{3}$ and an included angle of $\pi/6$.

106. Find the third side of a triangle with sides of lengths 7 and 10 and an included angle of 0.48277 radians. Retain all decimal places in the angle given, but round the answer to 4 decimal places.

107. Find the third side and the other two angles of an isosceles triangle with two sides of lengths 8 and an included angle of 81.083204°. Round your answers to four decimal places.

108. Find all three angles (each to the nearest one-hundredth of a degree) of the triangle with sides of lengths 16.0 m, 24.0 m, and 20.0 m.

     For each angle found, state the length of its opposite side.

     Then add the sizes of the three angles. Is the result as expected?

109. A central angle of 2° lies in a circle of radius 5 feet. To six decimal places, find the lengths of the arc and the chord determined by this angle.

     Which is longer? Does that agree with what is expected?

Additional Practice for the Final Examination

Math 130 Precalculus December 2011

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. Draw the unit circle. Designate its center by O.

   Plot all points on the unit circle for which the sine is equal to $\sqrt{3}/2$.

   (a) Call the points A and B.

      i. Connect them with the straight line segment AB. How long is this line segment?

      ii. How long is the arc of the unit circle between A and B?

      iii. Which is longer, the line segment or the arc? Which should be longer? Why?

   (b) Now connect the two points to the center of the circle, O, with line segments. How long is each line segment?

      i. Find the area of triangle AOB.

      ii. Find the area of the portion of the unit circle lying above the line segment AB.

   (c) For each of the two points A and B, find the $\sec^2$ and the $\tan^2$.

   For each point, how are the $\sec^2$ and the $\tan^2$ related? Of what well-known trig identity are these examples?

   (d) Find the magnitude of the central angle AOB in:

   - degrees.
   - radians.
   - revolutions.
   - grads.

   (e) Bisect the central angle AOB with a ray. At what point does this ray meet the unit circle?

2. An pellet is shot up from a rooftop so that its height above ground in meters at time t seconds when $t \geq 0$ and $y \geq 0$ is given by the equation

$$y = -5t^2 + 35t + 22.75.$$

   (a) Complete the square and graph, labelling with coordinates the vertex, and all t- and y-intercepts (even those where $t < 0$). Then graph the line of symmetry and label it with its equation.

   (b) How high is it initially? Does it ever get that high again. If so, when?

   (c) When is the object the highest? How far above the ground is it at that moment? When, if ever, does it reach that height again?

   (d) When, if ever, does it attain a height of 64 meters above the ground?

   (e) When does the object hit the ground?

3. Charlie Munger, an associate of Warren Buffett, invested \$16,000 in the financier's corporation in 1969. The money grew exponentially—assume it gained 22.3% in each and every year.

   Find the doubling time.

   Find the balance of Mr. Munger's investment in the year 2011.

   Find what the total value in 2011 would have been without reinvestment of profits, but the same 22.3% return. In that case, the total value will be represented by a linear function in the form $y = mt + b$.

   First, based on \$16,000, find the amount gained in the year from 1969 to 1970. Use those values of Δy and Δt to find m.

   For the total value in 2011, use Δt from 1969 to 2011 and m to get the change in y. Finally add the change in y to the original y, getting the total value.

4. Find its period and amplitude; then graph carefully a complete period of the graph of

$$y = 4\,\sin\left(x - \frac{\pi}{12}\right) + 1.$$

Plot and label with coordinates all x- and y-intercepts, all highest and lowest points, a point for which $y = -1$, and the point for which $x = \pi/4$.

5. Let $f(x) = \frac{1}{x} - x^2$.

Find and simplify $\dfrac{f(x+h) - f(x)}{h}$, assuming $h \neq 0$. (Answer is in terms of x and h.)

6. Let $f(x) = \dfrac{1}{x+2} + 2$.

Find a formula for f^{-1}, the inverse of f.

Find $f(-3)$, $f(-1)$, $f(0)$, $f^{-1}(1)$, $f^{-1}(2.5)$, and $f^{-1}(3)$. Then explain how these results validate your formula for f^{-1}.

7. Simplify:

$$2^4 + 2^{\log_4 81}, \qquad 2^4 \cdot 2^{\log_4 81}, \qquad 2^{4 - \log_4 81}, \qquad 2^{4 \div \log_{81} 4}.$$

8. Solve for x.
   (a) $\log(0.25 - x) + \log x = -2$.
   (b) $\log_8(6 - x) + \log_8 x = 1$.

9. Given that $\cos r = a$ with $0 < r < \pi/2$, and $\sin s = b$ with $\pi/2 < s < \pi$, find an expression for:
   (a) $\sin(r + s)$.
   (b) $\cos 2s$.
   (c) $\cos 2r$.
   (d) $\sin \frac{1}{2}r$.
   (e) $\sec \frac{1}{2}r$.

   The only letters in the answers should be a and b, as needed: no r or s.

10. A triangle has sides of 7 and 8, with an included angle of 120°. How long is the third side?

Sample of Typical Final Examination Problems

Math 130 Precalculus December 2011

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

For Problems 1 and 2:

Draw the unit circle. Designate its center by O.

Plot the highest point of the unit circle for which the cosine is equal to $-1/2$. Call it A.

Then plot the right-most point of the unit circle for which the sine and cosine are equal. Call it B.

1. Connect points A and B with a line segment (AB). Let the included arc of the circle be called arc AB.
   (a) i. Find, to 4 decimal places, the coordinates of the midpoint of line segment AB.
   ii. How long is this line segment?
   iii. How long is the arc?
   iv. Which is longer, the line segment or the arc? Which should be longer? Why?
   (b) Write the equation of the straight line passing through A and B. Round the slope to 5 decimals. Then plot the y-intercept of this line and label it with its coordinates (to 4-decimals).

2. (a) For point B, find the $\sec^2$ and the $\tan^2$.
   How are this $\sec^2$ and this $\tan^2$ related? Of what well-known trig identity are this an example?
   (b) Find the magnitude of the central angle AOB in:
   - degrees.
   - radians.
   - revolutions.
   - grads.

   (c) Bisect the central angle AOB with a ray. Let P be the intersection of the ray and the unit circle. Plot P and label it with its coordinates. You may express the coordinates either in terms of trig functions of a fractional multiple of π or as 4-place decimals: whichever form you choose.
   i. Let Q be the intersection of this ray and the line segment AB. Is Q the midpoint of line segment AB? Show—by comparing the coordinates of Q and of P—that answers are numerically reasonable and in agreement with the relative positions of the points in the plane.
   Will the coordinates of point Q—call them (a, b)—satisfy the equation $a^2 + b^2 = 1$?

3. An object is thrown up from a balcony so that its height above ground in feet at time t seconds when $t \geq 0$ and $y \geq 0$ is given by the equation

$$y = -16t^2 + 48t + 64.$$

   (a) Complete the square and graph, labelling with coordinates the vertex, and all t- and y-intercepts (even those where $t < 0$). Then graph the line of symmetry and label it with its equation.
   (b) How high is it initially? Does it ever get that high again. If so, when?
   (c) When is the object the highest? How far above the ground is it at that moment? When, if ever, does it reach that height again?
   (d) When, if ever, is it 125 feet above the ground? 75 feet above the ground?
   (e) When does the object hit the ground?

4. In China, there were 0.56 million users of the internet on January 1, 1996. The number of users grew exponentially pretty much at a constant rate with the percent of internet users increasing 58% each year.
   Based on the assumption that that rate will continue, find
   (a) the annual growth factor (multiplying the number of internet users in any year by that factor gives the number of internet users for the next year).
   (b) the doubling time.
   (c) the number of internet users on Janaury 1, 2108.
   (d) the first year in which the number of internet users in China will exceed 1 billion (1,000 million).

5. Find its period and amplitude; then graph carefully a complete period of the graph of

$$y = \frac{1}{2}\sin\left(x + \frac{\pi}{4}\right) + \frac{1}{2}.$$

Plot and label with coordinates all x- and y-intercepts, all highest and lowest points, a point for which $y = 3/4$, and the point for which $x = \pi$.

6. Let $f(x) = 3x^2 - 2x + 4$.

   Find and simplify an expression for $\dfrac{f(x+h) - f(x)}{h}$, assuming $h \neq 0$. (Answer is in terms of x and h.)

7. Let $f(x) = \dfrac{2x+3}{x-6} + 11$.

   Find a formula for f^{-1}, the inverse of f.

   Find $f(0)$, $f(1)$, $f^{-1}(10)$, and $f^{-1}(10\frac{1}{2})$. Then explain how these results validate your formula for f^{-1}.

8. Simplify:

$$6^{4+\log_{36} 9}, \qquad 6^{4\cdot\log_{36} 9}, \qquad 6^{4-\log_{36} 9}, \qquad 6^{4\div\log_{7} 6}.$$

9. Solve for x.
   (a) $\log_x\left(x^2 + \frac{3}{16}\right) = 1$.
   (b) $\log_8(5-x) + \log_8(x+1) = 1$.
   (c) $\log_{16}(4-x) + \log_{16}(x-3) = -\frac{1}{2}$.

10. Draw the first-quadrant portion of the unit circle. Designate the center of the circle as point O.
    (a) Plot and label with coordinates the following points:
        i. A, with cosine equal to $1/2$.
        ii. B, with sine equal to $\frac{1}{2}\sqrt{2}$.
        iii. C, with sine equal to $1/2$.
        iv. D, the midpoint of line segment AC.
        v. E, the midpoint of arc AC.

        Connect the points B and C with a line segment.
    (b) Now find the following:
        i. The size of the central angle BOC in degrees.
        ii. the coordinates of the midpoint of arc BC.
        iii. the length of arc BC.
        iv. the length of the line segment BC.
            A. Decide from these two lengths if the arc or the line segment is longer. Does this make sense?
        v. the coordinates of the midpoint of the line segment BC.
            A. Will these coordinates (call them (a,b)) satisfy the equation $a^2 + b^2 = 1$?
            B. Are the coordinates of the two midpoints numerically reasonable and in agreement with the relative positions of the points in the plane?

11. Given that $\pi/2 < x < \pi$ and $\sin x = a$, find—in terms of a—an expression for:
    (a) $\cos x$.
    (b) $\tan^2 x$.
    (c) $\sec^2 x$. (How is this related to $\tan^2 x$?)
    (d) $\sec 2x$.
    (e) $\sin \frac{1}{2}x$.
    (f) $\sin(x + \pi/2)$.

12. A triangle has sides of 6 and 10. The angle between them is 30°. Find the length of the third side.

Sample Final Examination

Math 130 Precalculus May 2009

Part A.

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. Plot the points A:$(3, 2.4)$ and B:$(13, -1.6)$; then connect them with a line segment.
   (a) Find the exact or four-decimal-place length of this line segment and the coordinates of its midpoint.
   (b) Find the equation of the line through these two points and find both intercepts. Plot these intercepts. Is the point $(8, 0.4)$ on the line? What about the point $(-7, 4.4)$?
   (c) Find the equation of the circle with center A that passes through B. Then roughly sketch the circle, labelling with coordinates all intercepts, the given points, and two other points on the circle. Find the coordinates of the rightmost point.

2. An object is thrown up so that its height above ground in feet at time t seconds when $t \geq 0$ and $y \geq 0$ is given by the equation
$$y = -16t^2 + 136t + 43.$$
   (a) Complete the square of that equation. Then graph it, label with coordinates the vertex and all t- and y-intercepts (even those where $t < 0$), and label the line of symmetry with its equation.
   (b) When is the object the highest? How far above the ground is it at that moment?
   (c) When will the height be 8 feet? 43 feet? When does the object hit the ground?

3. (a) Let $f(x) = \frac{1}{x} + 1$. Find and simplify an expression for $\frac{f(x+h) - f(x)}{h}$, assuming $h \neq 0$.
   (b) For same f: Let $x = 1$ and $h = 0.6$. Find $f(x+h)$ and $f(x)$. Then find $\frac{f(x+h) - f(x)}{h}$. Compare this with the result when 1 and 0.6 are substituted in the expression you previously found.

4. Of all right triangles for which the sum of the legs is 40 feet, the one with the largest area has three sides of what lengths? Find the area of that maximizing triangle. (You must find an appropriate function and maximize it.)

Part B.

5. (a) Simplify.
      i. $5^{4-\frac{1}{4}\log_5 4}$
      ii. $2 \cdot e^{\frac{1}{4}\ln(11e^2)}$
   (b) Solve for x. Then check your answers in the original equation.
      i. $\ln(e^3 - x^3) = \ln(e^2 - x^2) + 1$.
      ii. $\log_2(-x) + \log_2(6 - x) = 4$.
      iii. $\log x \cdot \ln x = 2 \log e$. (Hint: use change of base.)

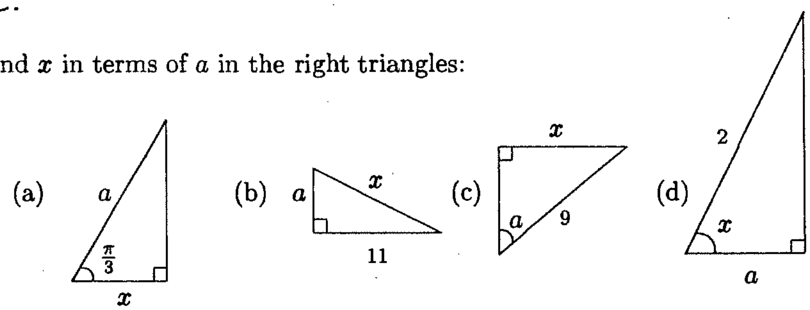
6. Assume that Citibank's toxic asset balance grows exponentially. At time $t = 3$ days it was \$30, but at time $t = 6$ days it was \$33.
   (a) Find an expression which gives the balance as a function of t in days.
   (b) What is the toxic asset balance at time $t = 63$ days?
   (c) When, if ever, is the balance \$300? \$3000? How are these two answers related?
   (d) Find the doubling time.

7. The temperature T in degrees Fahrenheit of a potato is given by the equation
$$T(t) = 425 - 355e^{-.06t},$$
   where t is the time in minutes after it is placed in the oven.
   (a) Graph $T(t)$ for $t \geq 0$, labelling with coordinates the points when $t = 0$, $t = 10$, $t = 60$, and $t = 600$, and graphing the asymptote and labelling it with its equation.
   (b) When, if ever is the temperature of that potato 350 degrees Fahrenheit?

TURN OVER

Part C.

8. Find x in terms of a in the right triangles:



9. A wheel of diameter 15 inches rolls 9 feet along the ground. As it does so:
   (a) How many revolutions does it make? Through how many degrees and radians does it turn?
   (b) If the wheel takes 2 seconds to roll 9 feet, what is its speed of revolution in:
   i) radians per second? *ii*) revolutions per minute?

10. (a) Graph $y = \arccos x$.
    Plot the points, labelling them with coordinates in four-place-decimal form, where
    i. $y = \sqrt{3}/2$
    ii. $y = -1/2$
    iii. $y = 1/4$
    iv. $x = 1/4$
    v. $x = \sqrt{2}/2$
    vi. $x = -\sqrt{3}/2$
    (b) Simplify $\arccos(\cos(-\pi/6))$. Explain why an "obvious" simplification does not give the correct answer.

11. Graph carefully two complete periods of the graph of
$$y = 2\sin(12\pi x) + 2.$$
Plot and label with coordinates all x- and y-intercepts, and all points where $y = 4$, $x = 1/48$, and $y = 3$.

Alternate questions

Do problems A, B, and C. Each will replace your worst score in A, B, or C.

A. Decide whether the function f with the given rule is even, odd, neither, or both and show why.
It might save time if you begin that investigation by finding $f(1)$ and $f(-1)$.
   (a) $f(x) = \sin x + \cos x$
   (b) $f(x) = \sqrt{x+1}$
   (c) $f(x) = x^3 - 2x^2 + x + 1$
   (d) $f(x) = \dfrac{2x^5 - x^3}{x + \sqrt[5]{x}}$
   (e) $f(x) = \dfrac{\sqrt[3]{x}}{x^2 + x^4}$

B. How is the graph of $y = \log_{64} x$ related to the graph of $y = \log_2 x$ through translation, stretching, or shrinking? Verify this by use of the change of base formula.
Name the points on each graph where $x = 8$ and show that their coordinates support that idea.
On each graph state the coordinates of the points where $y = 1/3$ and $y = -1/3$.

C. A triangle has sides of lengths 2, 2, and 3 feet.
   (a) Find the radian measure of the angle between the two sides of length 2 feet. Then find the radian measure of the other two angles.
   (b) Find the length of the arc of a circle with radius 2 that is subtended by the angle between the two sides of length 2 feet.
   (c) Find the length of the line segment that connects the midpoint of either side of length 2 feet to the opposite vertex.