1. **Definition**: A natural number $x$ is a **factor** of the natural number $y$, iff there is a natural number $c$, with $xc=y$.

2. **Definition**: A natural number is a **prime number**, iff it has exactly two factors, itself and 1.

3. **Remark**: The number 1 is NOT a prime number. (Why?)

4. Here is a list of the first few prime numbers:

| 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 | 71 |

5. Divisibility tricks: Divisible by 2 if last digit is even. Divisible by 3 if digits sum to a multiple of 3. No trick exists for 7. Trick for 11 is more complicated. Otherwise, you must divide.

6. **Theorem** (Euclid): The number of prime numbers is infinite.
   
   **Proof**: Suppose not. Then let $Q =$ the product of all possible prime numbers. Consider $Q+1$. $Q+1$ cannot be divisible by any of the prime numbers, because the remainder when dividing by that prime number would be 1 (not zero). Therefore, either there is another prime not in the list of primes, that is less than $Q+1$, that is a factor of $Q+1$; or else $Q+1$ is itself prime. Either way, the original list of primes was not complete. So there cannot be a finite number of primes.

7. **Definition**: If natural number $C$ is not prime, we say it **can be factored**.

8. **Remark**: Suppose $AB=C$. Then, either $A < B$ or $A = B$.

9. **Theorem**: Suppose $C$ is a natural number. If $C$ can be factored, then $C$ has a factor $A$ with $A \leq \sqrt{C}$.
   
   **Proof**: follows from the remark above.

10. **Algorithm**: **How to factor a natural number into prime factors**:

    If $C$ is a natural number, then we should try to divide $C$ by each prime number less than or equal to the square root of $C$. If one of those primes is a factor, then $C$ can be factored. Suppose $p_1$ is that factor. Let $C_1 = C/p_1$. Then continue as before, using $C_1$ in place of $C$. Continue until the result is prime. Write the result as a product of primes in ascending order, with exponents.

11. **Example**: Factor 323 into prime factors.

    We need to try the primes 2,3,5,7,11,13,17. Since $18^2 = 324$ (bigger than 323), we don't have to try any primes bigger than 17. After we divide each of these into 323, we find that $17(19) = 323$. Therefore $323 = 17 \cdot 19$.

12. **Example**: Factor 96 into prime factors.

    $96/2 = 48$. $48/2 = 24$. $24/2 = 12$. $12/2 = 6$. $6/2 = 3$. Therefore, $96 = 2^5 \cdot 3$.

13. **Example**: Factor 331 into prime factors.

    $19^2 = 361$ (greater than 329), so we don't need to try any primes bigger than 17. When we try the primes 2,3,5,7,11,13,17, we find that there is a non-zero remainder for each division. Therefore, 331 is a prime number.

14. **Exercise**: Factor these numbers into prime factors if possible:

    12, 28, 64, 100, 132, 327, 441, 1058, 1728.