A quadratic function is a function of the form \( y = ax^2 + bx + c \), where \( a \), \( b \), and \( c \) are real numbers.

Skill #1. Evaluating a quadratic function.
Example. \( f(x) = 3x^2 - 4x - 5 \). Evaluate \( f(x) \) for \( x = -2/3 \).
Answer: \( f(-2/3) = 3(-2/3)^2 - 4(-2/3) - 5 = 3(4/9) - 4(-2/3) - 5 = 4/3 + 8/3 - 15/3 = -3/3 = -1 \).

Skill #2. Calculating the discriminant.
A quadratic function has a discriminant \( \Delta = b^2 - 4ac \).
The discriminant of the function \( f(x) = 3x^2 - 4x - 5 \) is calculated as follows.
First write down \( a, b, \) and \( c \): Then write:
\[
\begin{array}{|c|c|}
\hline
a &= 3 \\
b &= -4 \\
c &= -5 \\
\hline
\end{array}
\]
\[
\Delta &= b^2 - 4ac \\
&= ( )^2 - 4 ( ) ( ) \\
&= (-4)^2 - 4 (3) (-5) \\
&= 16 + 60 = 76.
\]

Skill #3. Using the discriminant.
The discriminant \( \Delta \) tells you several things.
(a) If \( \Delta \) is a perfect square integer (\( \Delta = 0, 1, 4, 9, 16, 25 \), etc), then the quadratic polynomial factors over the integers.
(b) If \( \Delta > 0 \), then the quadratic function has two different real roots.
(c) If \( \Delta = 0 \), then the quadratic function has one real double root.
(d) If \( \Delta < 0 \), then the quadratic function has no real roots, but it has two complex conjugate roots.
Example: the polynomial function \( f(x) = 3x^2 - 4x - 5 \) has discriminant 76. Since 76 is not a perfect square, the polynomial cannot be factored over the integers. Since \( 76 > 0 \), the polynomial has two different real roots.

Skill #4. Does the parabola open up or down?
The graph of a quadratic function opens upwards (\( \cup \)) if the coefficient of \( x^2 \) is a positive number. The graph opens downwards (\( \cap \)) if the coefficient of \( x^2 \) is a negative number.

Skill #5. Finding the x-coordinate of the vertex, and the line of symmetry (axis of symmetry).
The vertex of a parabola (or quadratic function) is the \((x,y)\) point which is the highest or lowest point on the curve. The x-value of the vertex is half-way between the two roots, or the average of the two roots. It may be calculated directly by the formula \( x = -b/(2a) \).
Example: the x-value of the vertex of the equation \( f(x) = 3x^2 - 4x - 5 \) is \( x = -b/(2a) = -(-4)/(2(3)) = 4/6 = 2/3 \).
The line of symmetry (axis of symmetry) of the parabola is the vertical line through the vertex.
Skill #6. Finding the **y-coordinate of the vertex**.

The y-coordinate of the vertex is found by evaluating f(x) at the x-coordinate of the vertex. In the above example, the y-coordinate of the vertex is f(-b/(2a)) = f(2/3) = -19/3. (see Skill #1 above).

Skill #7. Finding the **roots** (also called the **x-intercepts**).

The roots of a function are the x-values for which the function value (y-value) is zero. Use factoring; but if the polynomial is not factorable, use the quadratic formula. The QF (quadratic formula) for the roots of a quadratic function may be stated as follows:

\[
\text{If } ax^2 + bx + c = 0, \text{ then } \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{\Delta}}{2a}.
\]

Before evaluating the QF, be sure to write the values of a, b, c (see skill #2). Note: from this formula, you can see that -b/(2a) is the average of the roots. Further, the distance between the roots is ± √Δ / a, and the distance along the x axis from either root to the line of symmetry is (√Δ) / (2a).

Skill #8. Finding the **y-intercept**.

Find the y-intercept by evaluating the polynomial at x=0.
In our example f(0) = 3(0)^2 - 4(0) - 5 = -5. The y-intercept is at the point (0,-5).

Skill #9. Finding the symmetric point.

The symmetric point is at the same height as the y-intercept point. It is symmetric about the axis of symmetry to the y-intercept; so it is the same distance from the axis of symmetry as the y-intercept: (y-intercept) -------- axis ------------ (symmetric point). So the symmetric point is a distance 2 (-b/2a) = -b/a from the y-axis.

Skill #10. Graphing the quadratic function.

When graphing, do all the above things in order.
Your sketch should show: (a) the roots; (b) the vertex; (c) the line of symmetry; (d) the y-intercept; (e) the symmetric point; and perhaps one or two other points.

Skill #11. Solving the quadratic by completing the square. [not discussed here].
Graphing quadratic functions.

Exercises. Graph the following quadratic functions. Calculate the roots, the vertex, and the discriminant. Find the y-intercept and the symmetric point. Show at least one or two other points on the graph.

<table>
<thead>
<tr>
<th>Equation</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Δ</th>
<th># of real roots</th>
<th>vertex</th>
<th>y-intercept</th>
<th>Symmetric point</th>
<th>roots</th>
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</thead>
<tbody>
<tr>
<td>1 (f(x) = x^2 - 6x + 8)</td>
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<td>2 (f(x) = x^2 + 4x + 3)</td>
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<td>3 (f(x) = x^2 - 2x - 15)</td>
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<td>4 (f(x) = x^2 + 2x - 8)</td>
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<td>5 (f(x) = -x^2 - 2x + 3)</td>
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<td>7 (f(x) = 2x^2 + 7x + 3)</td>
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<td>8 (f(x) = 3x^2 - 7x + 2)</td>
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<td>9 (f(x) = -4x^2 + 4x - 1)</td>
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<td>10 (f(x) = x^2 + 6x + 9)</td>
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<td>11 (f(x) = x^2 + 2x + 5)</td>
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<td>12 (f(x) = -2x^2 + 4x - 3)</td>
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<td>13 (f(x) = x^2 + 2x - 5)</td>
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<td>14 (f(x) = -x^2 + 4x - 1)</td>
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