Title : Spectral Sequences : some representation-theoretic applications

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ABSTRACT.

Spectral sequences were introduced by Jean Leray , in 1946 , in connection with problems in algebraic topology where a tool more powerful than exact sequences was required to relate , for example , various cohomology or homotopy groups . Their applications abound in many areas that range , for example , from algebraic geometry to BRST cohomology of 2-dimensional gravity (coupled to matter) . We present some expository remarks on spectral sequences with some concrete examples and applications in representation theory . For example , we consider (i) nil-radical Lie algebra cohomology with coefficients in the space of K-finite vectors of an irreducible unitary representation of a semisimple Lie group G with maximal compact subgroup K . The results here lead , for example , to (automorphic) multiplicity formulas for derived functor modules (representations constructed by the method of cohomological parabolic induction) , and in particular to a result conjectured by R.Langlands (in the discrete series case) and more generally to a solution of G.Warner ' s 3 rd problem (for representations of holomorphic type) – without recourse to the trace formula (ii) a BGG type resolution of holomorphic Verma modules that extends initial work of R.Stanke - i.e. a result for a general Hermitian G/K .