Unitary Realization of Cohomologically Induced Modules

1. Abstract

An open problem in representation theory is to find explicit geometric models for irreducible unitary representations of reductive groups. In this talk I will consider those representations "attached" to elliptic orbits. These are the cohomologically induced modules, $A_q(\lambda)$, in the title. The aim is to build the unitary globalization of the modules $A_q(\lambda)$ that are, by algebraic methods, known to be unitarizable. In many cases when such construction exists, the analytic-geometric construction uses the algebraic knowledge that the modules are unitarizable. In most cases the analytic-geometric construction of a Hilbert space that exhibits the module as unitary is not known, even when assuming unitarizability. In this talk I will survey the state of the problem, describe some of the methods that has been used indicating their partial successes. I will focus on the various geometric- analytic subtleties of the problem.