Instructor: Alfred Noël
Homework II
Due March 22

1. Compute the equation of the plane tangent to the surface defined by $3xy + z^2 = 4$ at (1,1,1).

2. The partial differential equation $u_t + u_{xxx} + uu_x = 0$, called the *Korteweg-de Vries* equation, describes the motion of water waves in a shallow channel. Show that for any positive constant c, the function

$$u(x,t) = (3c)\text{sech}^2[\frac{1}{2}(x - ct)\sqrt{c}]$$

is a solution of the KdV equation. This solution represents a traveling "hump" of water in the channel and is called a *soliton*.

3. Show that near the point $(x, y, u, v) = (1, 1, 1, 1)$ we can solve the system

$$xu + yvu^2 = 2$$
$$xu^3 + y^2v^4 = 2$$

uniquely for u and v as functions of x and y. Find $\frac{\partial u}{\partial x}(1, 1)$.

4. Do numbers 18, 20, 30, 34 in section 4.1

5. (a) Show that $(0, 0, 0)$ is a critical point of the function

$$f(x, y, z) = x^2 + xy + z^2 - \cos y.$$

(b) What kind of critical point is $(0, 0, 0)$?