Instructor: Alfred Noël
Homework III
Due April 19, 2005

1. Let $c_1(t) = (e^t, \sin t, t^3)$ and $c_2(t) = (e^{-t}, \cos t, -2t^3)$. Compute

$$\frac{d}{dt}(c_1 \times c_2)$$

using two different methods to verify the cross product rule for derivatives.

2. The cycloid is defined as $c(t) = (t - \sin t, 1 - \cos t)$. Find the velocity and the speed of the cycloid. Compute the length of one arc of the cycloid for $0 < t < 2\pi$.

3. Determine the moving frame (T, N, B) and compute the curvature and torsion for

$$X(t) = (\sin t - t\cos t, \cos t + t\sin t, 2)$$

4. Prove that for a curve $x(t)$ in $\mathbb{R}^3$ then its curvature can be computed using the following formula:

$$\kappa = \frac{||x' \times x''||}{||x'||^3}$$

Use the above formula to find the curvature of $(e^{-t}\cos t, e^{-t}\sin t, 0)$. What happens as $t \to \infty$

5. Consider the vector field $F = (2xye^z, e^z x^2, x^2ye^z + z^2)$.
   i. Compute div F and curl F
   ii. Find a function $f(x, y, z)$ such that $F = \nabla f$.

6. Show that $F = (x^2 + y^2)\vec{i} - (2xy)\vec{j}$ is not a gradient field.

7. Read pages 276-279 in the text book and do number 10 on page 280.