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Tables of Results

GI.

Let $\Delta = \{\alpha_1, \alpha_2\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \beta_2\}$, where $\beta_1 = \alpha_1$ and $\beta_2 = 3\alpha_1 + 2\alpha_2$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{sl}_2(\mathbb{C}) \oplus \mathfrak{sl}_2(\mathbb{C})$.

Table I

1. 1 1	$\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
	$x = H_{\alpha_1} + H_{\alpha_2}, \quad e = X_{3\alpha_1+\alpha_2}$
	$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + 3H_{\alpha_2})$
2. 1 3	$\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$
	$x = 2H_{\alpha_1} + 3H_{\alpha_2}, \quad e = X_{2\alpha_1+\alpha_2}$
	$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2}$

FI.

Let $\Delta = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \ldots, \beta_4\}$, where $\beta_1 = \alpha_4$, $\beta_2 = \alpha_3$, $\beta_3 = \alpha_2$ and $\beta_4 = 2\alpha_1 + 3\alpha_2 + 4\alpha_3 + 2\alpha_4$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{sp}_3(\mathbb{C}) \oplus \mathfrak{sl}_2(\mathbb{C})$.

Table II

6. 000 4	$\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2$
	$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 4H_{\alpha_3} + 2H_{\alpha_4}$, $e = \sqrt{2}(X_{\alpha_1} + X_{\alpha_1+3\alpha_2+4\alpha_3+2\alpha_4})$
	$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4}$
7. 200 0	$\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_1$
	$x = 2H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4}$, $e = \sqrt{2}(X_{-\alpha_1} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4})$
	$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}(2H_{\alpha_1} + 4H_{\alpha_2} + H_{\alpha_4})$

8. 002 2 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = 2H_{\alpha_1} + 6H_{\alpha_2} + 4H_{\alpha_3} + 2H_{\alpha_4}$

$e = X_{\alpha_1+\alpha_2+2\alpha_3} + X_{-\alpha_1} + X_{\alpha_1+2\alpha_2+3\alpha_3+2\alpha_4}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3}$

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9. 020 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = 4H_{\alpha_2} + 4H_{\alpha_3} + 2H_{\alpha_4}$

$e = \sqrt{2}(X_{\alpha_1+2\alpha_2+3\alpha_3+\alpha_4} + X_{-\alpha_1-\alpha_2-\alpha_3})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} + H_{\alpha_4})$

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19. 004 8 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = 8H_{\alpha_1} + 18H_{\alpha_2} + 12H_{\alpha_3} + 6H_{\alpha_4}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4} + X_{\alpha_1+\alpha_2+\alpha_3}) + \sqrt{10}X_{-\alpha_1}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3}$

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20. 204 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 4H_{\alpha_1} + 14H_{\alpha_2}10H_{\alpha_3} + 6H_{\alpha_4}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4} + X_{-\alpha_1-\alpha_2-\alpha_3}) + \sqrt{10}X_{\alpha_1+2\alpha_2+2\alpha_3}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3}$

FII.

Let $\Delta = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ be the Bourbaki system of simple roots of $\mathfrak{g}_{\mathbb{C}}$. Then $\Delta_k = \{\beta_1, \beta_2, \beta_3, \beta_4 : \beta_1 = \alpha_2 + 2\alpha_3 + 2\alpha_4, \beta_2 = \alpha_1, \beta_3 = \alpha_2, \beta_4 = \alpha_3, \}$ is a system of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{so}_4(\mathbb{C})$.

Table III

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2. 4000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2$

$x = 4H_{\alpha_1} + 8H_{\alpha_2} + 6H_{\alpha_3} + 4H_{\alpha_4}$, $e = \sqrt{2}(X_{\alpha_1+2\alpha_2+3\alpha_3+\alpha 4} + X_{\alpha_4})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_2}$

EI.

Let $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_6\}$ the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_0, \beta_4, \beta_3, \beta_2\}$, where $\beta_1 = \alpha_2$, $\beta_2 = \alpha_4$, $\beta_3 = \frac{\alpha_3+\alpha_5}{2}$, $\beta_4 = \frac{\alpha_1+\alpha_6}{2}$ and $\beta_0 = -(\beta_1+2\beta_2+3\beta_3+2\beta_4)$ is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = C_4$.

Table IV

4. 0002 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = -2H_{\alpha_2}$

$e = (X_{-\alpha_2-\alpha_3-\alpha_4} - \theta X_{-\alpha_2-\alpha_3-\alpha_4}) + (X_{\alpha_4+\alpha_5} - \theta X_{\alpha_4+\alpha_5})$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5})$

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5. 2000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T2$

$x = -2H_{\alpha_1} - 4H_{\alpha_2} - 4H_{\alpha_3} - 6H_{\alpha_4} - 4H_{\alpha_5} - 2H_{\alpha_6}$

$e = \sqrt{2}(X_{-\alpha_1-\alpha_2-2\alpha_3-3\alpha_4-2\alpha_5-\alpha_6} + X_{-\alpha_2})$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6}$

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6. 0200 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = -4H_{\alpha_2} - 2H_{\alpha_3} - 4H_{\alpha_4} - 2H_{\alpha_5}$

$e = \sqrt{2}(X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} - \theta X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} + X_{\alpha_1+\alpha_3+\alpha_4} - \theta X_{\alpha_1+\alpha_3+\alpha_4})$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5})$

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9. 0202 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = -6H_{\alpha_2} - 2H_{\alpha_3} - 4H_{\alpha_4} - 2H_{\alpha_5}$

$e = \sqrt{3}(X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} - \theta X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} + X_{\alpha_1} - \theta X_{\alpha_1})$
$+2(X_{\alpha_3+\alpha_4} - \theta X_{\alpha_3+\alpha_4})$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6})$

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12. 2002 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = -2H_{\alpha_1} - 6H_{\alpha_2} - 4H_{\alpha_3} - 6H_{\alpha_4} - 4H_{\alpha_5} - 2H_{\alpha_6}$

$e = X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5-\alpha_6} - \theta X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5-\alpha_6} + 2X_{-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{3}(X_{\alpha_3+\alpha_4} - \theta X_{\alpha_3+\alpha_4})$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_3} + H_{\alpha_5})$

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13. 2004 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = -2H_{\alpha_1} - 8H_{\alpha_2} - 4H_{\alpha_3} - 6H_{\alpha_4} - 4H_{\alpha_5} - 2H_{\alpha_6}$

$e = \sqrt{6}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5-\alpha_6} + \sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{6}(X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5} - \theta X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} + H_{\alpha_5} + H_{\alpha_6})$

EII.

Let $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_6\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \ldots, \beta_6\}$, where $\beta_1 = \alpha_1$, $\beta_2 = \alpha_3$, $\beta_3 = \alpha_4$, $\beta_4 = \alpha_5$, $\beta_5 = \alpha_6$ and $\beta_6 = \alpha_1 + 2\alpha_2 + 2\alpha_3 + 3\alpha_4 + 2\alpha_5 + \alpha_6$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{sl}_6(\mathbb{C}) \oplus \mathfrak{sl}_2(\mathbb{C})$.

Table V

6. 00000 4 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_2$

$x = 2H_{\alpha_1} + 4H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 4H_{\alpha_5} + 2H_{\alpha_6}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_3}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4})$

7. 20002 0 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1 + T_2$

$x = 2H_{\alpha_1} + 2H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + 2H_{\alpha_6}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_1} - H_{\alpha_6})$

8. 00200 2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_2$

$x = 2H_{\alpha_1} + 2H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 4H_{\alpha_5} + 2H_{\alpha_6}$

$e = X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_6})$

11. 02020 0 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1$

$x = 2H_{\alpha_1} + 4H_{\alpha_3} + 4H_{\alpha_4} + 4H_{\alpha_5} + 2H_{\alpha_6}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{-\alpha_2-\alpha_3-\alpha_4})$
$\quad + \sqrt{2}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_4-\alpha_5})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_5} + H_{\alpha_6})$

20. 00400 0 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$

$x = 2H_{\alpha_1} + 4H_{\alpha_3} + 6H_{\alpha_4} + 4H_{\alpha_5} + 2H_{\alpha_6}$

$e = \sqrt{4-u}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + \sqrt{u-2}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{6-u}X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$\quad + 2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{e}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_5})$

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21. 02020 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_2$

$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 8H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 4H_{\alpha_6}$

$e = \sqrt{4-u}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5} + \sqrt{u-2}X_{-\alpha_2-\alpha_4} + \sqrt{6-u}X_{-\alpha_2}$
$+2X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + \sqrt{e}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_5})$

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22. 20202 2 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 4H_{\alpha_1} + 2H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6}$

$e = X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + 2X_{-\alpha_2-\alpha_4}$
$+\sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3} - 2H_{\alpha_5} - H_{\alpha_6})$

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23. 00400 8 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2$

$x = 6H_{\alpha_1} + 8H_{\alpha_2} + 12H_{\alpha_3} + 18H_{\alpha_4} + 12H_{\alpha_5} + 6H_{\alpha_6}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4} + X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5} + X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6}) + \sqrt{10}X_{-\alpha_2}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_6})$

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24. 20402 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_1$

$x = 6H_{\alpha_1} + 4H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 10H_{\alpha_5} + 6H_{\alpha_6}$

$e = \sqrt{10}X_{+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_4} + X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_5})$

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25. 40004 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_1$

$x = 6H_{\alpha_1} + 4H_{\alpha_2} + 8H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6})$
$+2(X_{-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} + X_{-\alpha_2})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_5} - H_{\alpha_6})$

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26. 22022 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_2$

$x = 4H_{\alpha_1} + 6H_{\alpha_3} + 6H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6}$

$e = 2(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{6}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_6})$

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34. 22422 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 8H_{\alpha_1} + 4H_{\alpha_2} + 14H_{\alpha_3} + 18H_{\alpha_4} + 14H_{\alpha_5} + 8H_{\alpha_6}$

$e = \sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + 2X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$+\sqrt{6}(X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} + X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6})$

$\mathfrak{k}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} - H_{\alpha_3} + H_{\alpha_5} - H_{\alpha_6})$

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35. 40404 8 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 10H_{\alpha_1} + 8H_{\alpha_2} + 16H_{\alpha_3} + 22H_{\alpha_4} + 16H_{\alpha_5} + 10H_{\alpha_6}$

$e = \sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4} + \sqrt{8}X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + \sqrt{10}X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{14}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + \sqrt{18}X_{-\alpha_2-\alpha_4}$

$\mathfrak{k}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} - H_{\alpha_3} + H_{\alpha_5} - H_{\alpha_6})$

EIII.

Let $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_6\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \ldots, \beta_6\}$, where $\beta_1 = \alpha_1$, $\beta_2 = \alpha_3$, $\beta_3 = \alpha_4$, $\beta_4 = \alpha_2$, $\beta_5 = \alpha_5$ and $\beta_6 = -\alpha_1 - 2\alpha_2 - 2\alpha_3 - 3\alpha_4 - 2\alpha_5 - \alpha_6$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{so}_{10}(\mathbb{C}) \oplus \mathbb{C}$.

Table VI

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6. 02000 -2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_2 + A_1 + T_1$

$x = 2H_{\alpha_1} + 2H_{\alpha_2} + 4H_{\alpha_3} + 4H_{\alpha_4} + 2H_{\alpha_5}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_6})$

$\mathfrak{k}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}H_{\alpha_4}$

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9. 40000 -2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq G_2$

$x = 4H_{\alpha_1} + 2H_{\alpha_2} + 4H_{\alpha_3} + 4H_{\alpha_4} + 2H_{\alpha_5}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6})$
$+\sqrt{2}(X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6} + X_{-\alpha_6})$

$\mathfrak{k}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4}$

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12. 02022 -6 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$

$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 10H_{\alpha_4} + 6H_{\alpha_5}$

$e = +\sqrt{6}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_3-\alpha_4-\alpha_5-\alpha_6})$
$+2(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6})$

$\mathfrak{k}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(2H_{\alpha_1} + H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_6})$

EIV.

Let $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_6\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \ldots, \beta_4\}$, where $\beta_1 = \alpha_2$, $\beta_2 = \alpha_4$, $\beta_3 = \frac{\alpha_3+\alpha_5}{2}$ and $\beta_4 = \frac{\alpha_1+\alpha_6}{2}$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = F_4$.

Table VII

2. 0002 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2$

$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6}$

$e = \sqrt{2}(X_{\alpha_1} - \theta X_{\alpha_1} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} - \theta X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5})$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4}$

EV.

Let $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_7\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \ldots, \beta_7\}$, where $\beta_1 = \alpha_1$, $\beta_2 = \alpha_3$, $\beta_3 = \alpha_4$, $\beta_4 = \alpha_5$, $\beta_5 = \alpha_6$, $\beta_6 = \alpha_7$ and $\beta_7 = \alpha_1 + 2\alpha_2 + 2\alpha_3 + 3\alpha_4 + 2\alpha_5 + \alpha_6$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{sl}_8(\mathbb{C})$.

Table VIII

3. 0200000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq C_3 + A_1$

$x = 2H_{\alpha_1} + H_{\alpha_2} + 4H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7}$

$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_2}$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6})$

4. 000020 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq C_3 + A_1$

$x = 2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + 3H_{\alpha_7}$

$e = X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_5})$

6. 2000002 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_2 + T_1$

$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6})$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_7})$

7. 0002000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3$

$x = 2H_{\alpha_1} + 2H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$

$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_1} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_3} - H_{\alpha_6})$

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16. 4000000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2$

$x = 4H_{\alpha_1} + H_{\alpha_2} + 4H_{\alpha_3} + 4H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7})$
$+X_{-\alpha_2-\alpha_4-\alpha_5} + X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + X_{-\alpha_2-\alpha_3-2\alpha_4-\alpha_5-\alpha_6-\alpha_7}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_3} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_4} - H_{\alpha_7})$

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17. 0000004 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2$

$x = 4H_{\alpha_1} + 7H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 9H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_2+\alpha_4} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}) + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5})$

---

18. 2000200 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = 4H_{\alpha_1} + 3H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 7H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_2} + X_{-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_3} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_7})$

---

19. 0020002 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = 4H_{\alpha_1} + 5H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 9H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} - H_{\alpha_6}) \oplus \mathbb{C}(2H_{\alpha_1} + H_{\alpha_5} + H_{\alpha_7})$

---

21. 0200020 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$

$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 8H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6} + 4H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} - H_{\alpha_6})$
$\oplus \mathbb{C}(2H_{\alpha_2} + H_{\alpha_3} + 4H_{\alpha_4} + H_{\alpha_7})$

---

22. 0202000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$

$x = 4H_{\alpha_1} + 3H_{\alpha_2} + 8H_{\alpha_3} + 10H_{\alpha_4} + 9H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$

$e = \sqrt{3}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2+-\alpha_3-\alpha_4})$
$+2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_5})$

---

23. 0002020 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 3A_1$

$x = 4H_{\alpha_1} + 5H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 11H_{\alpha_5} + 8H_{\alpha_6} + 5H_{\alpha_7}$

$e = 2X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_4} + H_{\alpha_5})$

---

26. 2002002 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$

$x = 6H_{\alpha_1} + 6H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 12H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$

$e = X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{3}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4}) + 2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_7})$

---

27. 0020200 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_3$

$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$

$e = \sqrt{4-u}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + 2X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{2-u}X_{-\alpha_2-\alpha_4}$
$+\sqrt{u}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{2+u}X_{-\alpha_2-\alpha_4-\alpha_5}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_1} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4} - H_{\alpha_6})$

---

30. 2004002 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_2 + T_1$

$x = 8H_{\alpha_1} + 8H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 18H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7})$
$+\sqrt{10}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_3} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_4} - H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5})$

---

38. 2200022 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_2$

$x = 8H_{\alpha_1} + 8H_{\alpha_2} + 14H_{\alpha_3} + 18H_{\alpha_4} + 14H_{\alpha_5} + 10H_{\alpha_6} + 6H_{\alpha_7}$

$e = 2(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6})$
$+\sqrt{6}(X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} - H_{\alpha_7})$

---

39. 0040000 $\mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1$

$x = 4H_{\alpha_1} + 3H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 9H_{\alpha_5} + 6H_{\alpha_6} + 3H_{\alpha_7}$

$e = \sqrt{2}X_{+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + 2X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{2}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{3}(X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} + H_{\alpha_5} - H_{\alpha_6} - 2H_{\alpha_7})$

---

40. 0040000 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1$

$x = 4H_{\alpha_1} + 5H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 11H_{\alpha_5} + 10H_{\alpha_6} + 5H_{\alpha_7}$

$e = \sqrt{2}X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6} + 2X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{3}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + \sqrt{2}X_{-\alpha_2-\alpha_4} + \sqrt{3}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(2H_{\alpha_2} + 3H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6})$

---

41. 2020020 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 6H_{\alpha_1} + 5H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 11H_{\alpha_5} + 8H_{\alpha_6} + 5H_{\alpha_7}$

$e = X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{3}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{2}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+2X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{3}X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + \sqrt{2}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_2} + H_{\alpha_3} + 3H_{\alpha_4} - H_{\alpha_6})$

---

42. 0200202 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 6H_{\alpha_1} + 7H_{\alpha_2} + 12H_{\alpha_3} + 16H_{\alpha_4} + 13H_{\alpha_5} + 10H_{\alpha_6} + 5H_{\alpha_7}$

$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + \sqrt{3}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{2}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{2} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{3}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + 2X_{-\alpha_2-\alpha_4}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_7})$

---

43. 0202020 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1$

$x = 6H_{\alpha_1} + 6H_{\alpha_2} + 12H_{\alpha_3} + 16H_{\alpha_4} + 14H_{\alpha_5} + 10H_{\alpha_6} + 6H_{\alpha_7}$

$e = \sqrt{3}(X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}) + 2X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_4-\alpha_5}) + 2X_{-\alpha_2-\alpha_3-\alpha_4}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_6})$

---

44. 0402020 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1$

$x = 8H_{\alpha_1} + 7H_{\alpha_2} + 16H_{\alpha_3} + 20H_{\alpha_4} + 17H_{\alpha_5} + 12H_{\alpha_6} + 7H_{\alpha_7}$

$e = \sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{5}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6})$
$+\sqrt{8}X_{+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{5}X_{-\alpha_2-\alpha_3-\alpha_4} + 3X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5})$

---

45. 0202040 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1$

$x = 8H_{\alpha_1} + 9H_{\alpha_2} + 16H_{\alpha_3} + 22H_{\alpha_4} + 19H_{\alpha_5} + 14H_{\alpha_6} + 9H_{\alpha_7}$

$e = 3X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + \sqrt{5}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{8}X_{-\alpha_2-\alpha_4-\alpha_5} + \sqrt{5}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_5} + H_{\alpha_6})$

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54. 2020202 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 8H_{\alpha_1} + 8H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$

$e = 2(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}) + \sqrt{2}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{6}(X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_3-\alpha_4}) + \sqrt{2}X_{-\alpha_2-\alpha_4-\alpha_5}$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_5} + 2H_{\alpha_6} + 3H_{\alpha_7})$

---

55. 4004000 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = 8H_{\alpha_1} + 5H_{\alpha_2} + 12H_{\alpha_3} + 16H_{\alpha_4} + 15H_{\alpha_5} + 10H_{\alpha_6} + 5H_{\alpha_7}$

$e = \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5}+\sqrt{7}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}+\sqrt{7-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{u-2}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}+\sqrt{12-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}+X_{-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{u}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_3} - H_{\alpha_7})$

---

56. 0004004 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = 8H_{\alpha_1} + 11H_{\alpha_2} + 16H_{\alpha_3} + 24H_{\alpha_4} + 21H_{\alpha_5} + 14H_{\alpha_6} + 7H_{\alpha_7}$

$e = \sqrt{7-u}X_{\alpha_2+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{7}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + \sqrt{5-u}X_{-\alpha_2} + \sqrt{5+u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_3} + H_{\alpha_6} + H_{\alpha_7})$

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57. 2022020 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 8H_{\alpha_1} + 7H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 17H_{\alpha_5} + 12H_{\alpha_6} + 7H_{\alpha_7}$

$e = \sqrt{12-u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}+\sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}+X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{7}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6} + \sqrt{u-5}X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + \sqrt{10-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$+\sqrt{u}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_3} + H_{\alpha_6})$

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58. 0202202 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 8H_{\alpha_1} + 9H_{\alpha_2} + 16H_{\alpha_3} + 22H_{\alpha_4} + 19H_{\alpha_5} + 14H_{\alpha_6} + 7H_{\alpha_7}$

$e = \sqrt{7-u}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}+\sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}+\sqrt{5+u}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+X_{-\alpha_2-\alpha_4-\alpha_5} + \sqrt{10}X_{-\alpha_2-\alpha_3-\alpha_4} + \sqrt{u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{2-u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$

$\mathfrak{t}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_4} - H_{\alpha_7})$

---

62. 2202022 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 10H_{\alpha_1} + 10H_{\alpha_2} + 18H_{\alpha_3} + 24H_{\alpha_4} + 20H_{\alpha_5} + 14H_{\alpha_6} + 8H_{\alpha_7}$

$e = X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5} + \sqrt{8}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + 3X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{5}(X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4}) + \sqrt{8}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_7})$

---

63. 0220220 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 8H_{\alpha_1} + 8H_{\alpha_2} + 16H_{\alpha_3} + 22H_{\alpha_4} + 18H_{\alpha_5} + 14H_{\alpha_6} + 8H_{\alpha_7}$

$e = \sqrt{5}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + \sqrt{8}(X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7})$
$+3X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_2-\alpha_3-\alpha_4} + \sqrt{5}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_7})$

---

66. 2204022 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$

$x = 12H_{\alpha_1} + 12H_{\alpha_2} + 22H_{\alpha_3} + 30H_{\alpha_4} + 26H_{\alpha_5} + 18H_{\alpha_6} + 10H_{\alpha_7}$

$e = \sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + \sqrt{18}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{10}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{14}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} + \sqrt{10}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4} - H_{\alpha_7})$

---

71. 2220222 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 12H_{\alpha_1} + 12H_{\alpha_2} + 22H_{\alpha_3} + 30H_{\alpha_4} + 24H_{\alpha_5} + 18H_{\alpha_6} + 10H_{\alpha_7}$

$e = \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + \sqrt{10}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{12}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{12}X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5} + \sqrt{10}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} - H_{\alpha_5} - H_{\alpha_7})$

---

80. 4220224 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 16H_{\alpha_1} + 16H_{\alpha_2} + 28H_{\alpha_3} + 38H_{\alpha_4} + 30H_{\alpha_5} + 22H_{\alpha_6} + 12H_{\alpha_7}$

$e = \sqrt{8+u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4} + \sqrt{8-u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5} + \sqrt{22-u}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{12}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6} + \sqrt{24}X_{-\alpha_2-\alpha_3-\alpha_4} + \sqrt{u}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{12}X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_7})$

---

84. 4224224 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 20H_{\alpha_1} + 20H_{\alpha_2} + 36H_{\alpha_3} + 50H_{\alpha_4} + 42H_{\alpha_5} + 30H_{\alpha_6} + 16H_{\alpha_7}$

$e = \sqrt{42}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5} + \sqrt{30}X_{\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+4X_{\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{22}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4} + \sqrt{30}X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$+4X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_7})$

EVI.

Let $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_7\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \ldots, \beta_7\}$, where $\beta_1 = \alpha_7$, $\beta_2 = \alpha_6$, $\beta_3 = \alpha_5$, $\beta_4 = \alpha_4$, $\beta_5 = \alpha_3$, $\beta_6 = \alpha_2$ and $\beta_7 = 2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{so}_{12}(\mathbb{C})\oplus\mathfrak{sl}_2(\mathbb{C})$.

Table IX

6. 000000 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_5$

$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_6})$

---

7. 000020 2 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq C_3$

$x = 2H_{\alpha_1} + 4H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$

$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7}$

---

8. 020000 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3 + A_1 + T_1$

$x = 2H_{\alpha_2} + 2H_{\alpha_3} + 4H_{\alpha_4} + 4H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3} + H_{\alpha_6})$

---

14. 400000 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2 + A_1$

$x = 2H_{\alpha_2} + 2H_{\alpha_3} + 4H_{\alpha_4} + 4H_{\alpha_5} + 4H_{\alpha_6} + 4H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+\sqrt{2}(X_{-\alpha_1} + X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 4H_{\alpha_6} + H_{\alpha_7})$

---

15. 000200 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1 + T_1$

$x = 4H_{\alpha_2} + 4H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+\sqrt{2}(X_{-\alpha_1-\alpha_3-\alpha_4} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_5} + 2H_{\alpha_6}) \oplus \mathbb{C}(-H_{\alpha_2} + H_{\alpha_5} + H_{\alpha_7})$

---

19. 000040 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 3A_1$

$x = 4H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$

$e = 2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + \sqrt{2-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+\sqrt{4-u}X_{-\alpha_1-\alpha_3}+\sqrt{2+u}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}+\sqrt{u}X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7}$

---

20. 000200 4 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 3A_1$

$x = 4H\alpha_1 + 8H_{\alpha_2} + 10H_{\alpha_3} + 16H_{\alpha_4} + 12H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$

$e = \sqrt{2-u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5} + \sqrt{2+u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$
$+2X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{4-u}X_{-\alpha_1} + \sqrt{u}X_{-\alpha_1-\alpha_3}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7}$

---

21. 020020 2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1$

$x = 2H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$

$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6})$
$+2X_{-\alpha_1-\alpha_3} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_5} - H_{\alpha_7})$

---

22. 000040 8 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq C_3$

$x = 8H_{\alpha_1} + 12H_{\alpha_2} + 18H_{\alpha_3} + 24H_{\alpha_4} + 18H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7})$
$+\sqrt{10}X_{-\alpha_1} + \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_6} + H_{\alpha_7})$

---

23. 020040 4 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B2 + A_1$

$x = 4H_{\alpha_1} + 10H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$

$e = \sqrt{10}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+\sqrt{6}X_{-\alpha_1-\alpha_3} + X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7}$

---

25. 040000 4 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_2 + T_1$

$x = 4H_{\alpha_1} + 8H_{\alpha_2} + 10H_{\alpha_3} + 16H_{\alpha_4} + 14H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+2(X_{-\alpha_1-\alpha_3-\alpha_4} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_4} - H_{\alpha_7})$

---

26. 020200 0 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_2$

$x = 6H_{\alpha_2} + 6H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$

$e = 2X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+2X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} + \sqrt{6}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6})$

---

29. 004000 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = 6H_{\alpha_2} + 6H_{\alpha_3} + 12H_{\alpha_4} + 12H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$

$e = \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + 2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{2}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{6}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$+2X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + \sqrt{2}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5-\alpha_6-\alpha_7}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(2H_{\alpha_1} + 3H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 3H_{\alpha_5} - H_{\alpha_7})$

---

31. 020220 2 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = 2H_{\alpha_1} + 10H_{\alpha_2} + 12H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$

$e = \sqrt{5}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+3X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{8}(X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} + H_{\alpha_5} - H_{\alpha_7})$

---

32. 000400 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = 4H_{\alpha_1} + 12H_{\alpha_2} + 14H_{\alpha_3} + 24H_{\alpha_4} + 18H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$

$e = \sqrt{5}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+3X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{8}(X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$+X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}) + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} - H_{\alpha_5} + H_{\alpha_7})$

---

33. 020240 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = 4H_{\alpha_1} + 14H_{\alpha_2} + 18H_{\alpha_3} + 28H_{\alpha_4} + 22H_{\alpha_5} + 16H_{\alpha_6} + 8H_{\alpha_7}$

$e = \sqrt{18}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{8}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{10}(X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7})$
$+\sqrt{14}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_7})$

---

34. 040040 8 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = 8H_{\alpha_1} + 16H_{\alpha_2} + 22H_{\alpha_3} + 32H_{\alpha_4} + 26H_{\alpha_5} + 20H_{\alpha_6} + 10H_{\alpha_7}$

$e = \sqrt{10}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6})$
$+\sqrt{8}X_{-\alpha_1-\alpha_3-\alpha_4} + \sqrt{18}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$+\sqrt{14}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_2} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_5} - H_{\alpha_7})$

---

35. 400400 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = 10H_{\alpha_2} + 10H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7}$

$e = \sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+\sqrt{10}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + \sqrt{12}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5}$

$+\sqrt{12}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}+\sqrt{6}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6}$

$\mathfrak{k}^1_{\mathbb{C}}=\mathbb{C}(2H_{\alpha_1}+3H_{\alpha_2}+4H_{\alpha_3}+4H_{\alpha_4}+3H_{\alpha_5}+4H_{\alpha_6}+H_{\alpha_7})$

---

36. 040400 4 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq T_1$

$x=4H_{\alpha_1}+16H_{\alpha_2}+18H_{\alpha_3}+32H_{\alpha_4}+26H_{\alpha_5}+20H_{\alpha_6}+10H_{\alpha_7}$

$e=\sqrt{22}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}+\sqrt{22-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{12}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}+\sqrt{8+u}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}+\sqrt{12}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$+\sqrt{8+u}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5}$

$\mathfrak{k}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_2}-H_{\alpha_5}+H_{\alpha_7})$

---

37. 040440 8 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq A_1$

$x=8H_{\alpha_1}+24H_{\alpha_2}+30H_{\alpha_3}+48H_{\alpha_4}+38H_{\alpha_5}+28H_{\alpha_6}+14H_{\alpha_7}$

$e=\sqrt{22}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5}+\sqrt{30}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+4X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}+\sqrt{42}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{30}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}+4X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$

$\mathfrak{k}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_2}-H_{\alpha_5}+H_{\alpha_7})$

EVII.

Let $\Delta=\{\alpha_1,\alpha_2,\ldots,\alpha_7\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k=\{\beta_1,\ldots,\beta_7\}$, where $\beta_1=\alpha_6$, $\beta_2=\alpha_2$, $\beta_3=\alpha_5$, $\beta_4=\alpha_4$, $\beta_5=\alpha_3$, $\beta_6=\alpha_1$ and $\beta_7=-2\alpha_1-2\alpha_2-3\alpha_3-4\alpha_4-3\alpha_5-2\alpha_6-\alpha_7$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}}=E_6\oplus(\mathbb{C})$.

Table X

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6. 000000 2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq F_4$

$x=-(2H_{\alpha_1}+3H_{\alpha_2}+4H_{\alpha_3}+6H_{\alpha_4}+5H_{\alpha_5}+4H_{\alpha_6}+3H_{\alpha_7})$

$e=X_{-\alpha_1}+X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7}+X_{-2\alpha_1-2\alpha_2-3\alpha_3-4\alpha_4-3\alpha_5-2\alpha_6-\alpha_7}$

$\mathfrak{k}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_2}\oplus\mathbb{C}H_{\alpha_3}\oplus\mathbb{C}H_{\alpha_4}\oplus\mathbb{C}H_{\alpha_5}$

---

7. 000000 -2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq F_4$

$x=2H_{\alpha_1}+3H_{\alpha_2}+4H_{\alpha_3}+6H_{\alpha_4}+5H_{\alpha_5}+4H_{\alpha_6}+3H_{\alpha_7}$

$e=X_{\alpha_1}+X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}+X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{k}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_2}\oplus\mathbb{C}H_{\alpha_3}\oplus\mathbb{C}H_{\alpha_4}\oplus\mathbb{C}H_{\alpha_5}$

---

8. 000002 -2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq B_4$

$x = 2H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} + -H_{\alpha_7}$

$e = X_{-\alpha_7} + X_{-(\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7)} + X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5}$

---

9. 200000 -2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq b_4$

$x = 2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} + H_{\alpha_7}$

$e = X_{-\alpha_7} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5}$

---

10. 020000 -2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_4 + T_1$

$x = 2H_{\alpha_1} + 4H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 4H_{\alpha_5} + 2H_{\alpha_6}$

$e = \sqrt{2}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_7})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_2} + 2H_{\alpha_6} + H_{\alpha_7})$

---

15. 200002 -4 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq G_2 + T_1$

$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_6-\alpha_7}$
$+X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7} + X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_3} + H_{\alpha_5} + H_{\alpha_7})$

---

16. 200002 -2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_3$

$x = 2H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_5} - 3H_{\alpha_7}$

$e = 2X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + \sqrt{3}(X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5-\alpha_6-\alpha_7}+$
$X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7}) + X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5})$

---

17. 400000 -2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_3$

$x = 2H_{\alpha_1} + 3H_{\alpha_2} + 4H_{\alpha_3} + 6H_{\alpha_4} + 5H_{\alpha_5} + 4H_{\alpha_6} - H_{\alpha_7}$

$e = 2X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}+$
$\sqrt{3}(X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + X_{-\alpha_1-2\alpha_2-2\alpha_3-3\alpha_4-2\alpha_5-\alpha_6-\alpha_7})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5}$

---

18. 000004 -6 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_3$

$x = 6H_{\alpha_1} + 5H_{\alpha_2} + 8H_{\alpha_3} + 10H_{\alpha_4} + 7H_{\alpha_5} + 4H_{\alpha_6} + H_{\alpha_7}$

$e = \sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7})+$
$2X_{-\alpha_7} + X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5}$

19. 200002 -6 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_3$

$x = 6H_{\alpha_1} + 7H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 11H_{\alpha_5} + 8H_{\alpha_6} + 3H_{\alpha_7}$

$e = \sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7})+$
$2X_{-\alpha_7} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5})$

---

20. 220002 -6 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2 + T1$

$x = 6H_{\alpha_1} + 8H_{\alpha_2} + 10H_{\alpha_3} + 14H_{\alpha_4} + 10H_{\alpha_5} + 6H_{\alpha_6}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7})+$
$2(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_2} - 2H_{\alpha_3} + 2H_{\alpha_6} + H_{\alpha_7})$

---

21. 400004 -6 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2$

$x = 6H_{\alpha_1} + 5H_{\alpha_2} + 8H_{\alpha_3} + 10H_{\alpha_4} + 7H_{\alpha_5} + 4H_{\alpha_6} - 3H_{\alpha_7}$

$e = \sqrt{8}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7})+$
$\sqrt{5}(X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5-\alpha_6-\alpha_7} + X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7})$
$+3X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4}$

---

22. 400004 -10 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2$

$x = 10H_{\alpha_1} + 11H_{\alpha_2} + 16H_{\alpha_3} + 22H_{\alpha_4} + 17H_{\alpha_5} + 12H_{\alpha_6} + 3H_{\alpha_7}$

$e = \sqrt{5}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7})+$
$\sqrt{8}(X_{-\alpha_6-\alpha_7} + X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7})$
$+3X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4})$

EVIII.

Let $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_8\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \ldots, \beta_8\}$, where $\beta_1 = 2\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4 + 3\alpha_5 + 2\alpha_6 + \alpha_7$, $\beta_2 = \alpha_8$, $\beta_3 = \alpha_7$, $\beta_4 = \alpha_6$, $\beta_5 = \alpha_5$, $\beta_6 = \alpha_4$, $\beta_7 = \alpha_2$ and $\beta_8 = \alpha_3$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = \mathfrak{so}_{16}(\mathbb{C})$.

Table XI

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4. 02000000 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_5 + A_1$

$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 8H_{\alpha_6} + 6H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_4}) \oplus C(H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_5} + H_{\alpha_7})$

---

5. 00000020 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq C_4$

$x = 2H_{\alpha_1} + 6H_{\alpha_2} + 6H_{\alpha_3} + 10H_{\alpha_4} + 8H_{\alpha_5} + 6H_{\alpha_6} + 4H_{\alpha_7} + 2H_{\alpha_8}$

$e = X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_1}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}H_{\alpha_8} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_5} + H_{\alpha_7})$

---

14. 40000000 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2G_2$

$x = 8H_{\alpha_1} + 10H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{2}(X_{\alpha_1} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$+X_{\alpha_1+3\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}H_{\alpha_8}$

---

15. 20000002 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq G_2$

$x = 6H_{\alpha_1} + 10H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8})$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_5} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_2} - H_{\alpha_5} - H_{\alpha_6})$

---

16. 00020000 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 4A_1$

$x = 4H_{\alpha_1} + 8H_{\alpha_2} + 10H_{\alpha_3} + 16H_{\alpha_4} + 14H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_1})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_4} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_3} + 3H_{\alpha_4} + H_{\alpha_6})$

---

19. 02000020 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_2 + A_1$

$x = 6H_{\alpha_1} + 12H_{\alpha_2} + 14H_{\alpha_3} + 22H_{\alpha_4} + 18H_{\alpha_5} + 14H_{\alpha_6} + 10H_{\alpha_7} + 6H_{\alpha_8}$

$e = \sqrt{3}(X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7})$
$+2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_6} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_5})$

---

20. 00000200 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 4A_1$

$x = 4H_{\alpha_1} + 10H_{\alpha_2} + 12H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{4-u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + 2X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{u}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}+\sqrt{2-u}X_{-\alpha_1-\alpha_3-\alpha_4}+\sqrt{2+u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_6} + \mathbb{C}H_{\alpha_8} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_7})$

---

21. 02000040 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq C_3 + A_1$

$x = 8H_{\alpha_1} + 18H_{\alpha_2} + 20H_{\alpha_3} + 32H_{\alpha_4} + 26H_{\alpha_5} + 20H_{\alpha_6} + 14H_{\alpha_7} + 8H_{\alpha_8}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}) + \sqrt{10}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_5} + \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_4} + H_{\alpha_6})$

---

29. 02020000 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_2 + A_1 + T_1$

$x = 8H_{\alpha_1} + 14H_{\alpha_2} + 18H_{\alpha_3} + 28H_{\alpha_4} + 24H_{\alpha_5} + 20H_{\alpha_6} + 14H_{\alpha_7} + 8H_{\alpha_8}$

$e = 2(X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8})$
$+\sqrt{6}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_5} + H_{\alpha_6})$

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30. 00020020 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_2$

$x = 6H_{\alpha_1} + 14H_{\alpha_2} + 16H_{\alpha_3} + 26H_{\alpha_4} + 22H_{\alpha_5} + 18H_{\alpha_6} + 12H_{\alpha_7} + 6H_{\alpha_8}$

$e = 2(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8})$
$+\sqrt{3}(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 3H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_4} - H_{\alpha_7})$

---

34. 00000004 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_2$

$x = 4H_{\alpha_1} + 10H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{2-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5} + \sqrt{2+u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6}$
$+\sqrt{2}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + \sqrt{4-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5}$
$\sqrt{2}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + 2X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(2H_{\alpha_1}+4H_{\alpha_2}+3H_{\alpha_3}+7H_{\alpha_4}+4H_{\alpha_5}+3H_{\alpha_6}+2H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_2}+H_{\alpha_4}-H_{\alpha_8})$

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35. 200020002 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$

$x = 8H_{\alpha_1} + 14H_{\alpha_2} + 18H_{\alpha_3} + 28H_{\alpha_4} + 24H_{\alpha_5} + 18H_{\alpha_6} + 12H_{\alpha_7} + 6H_{\alpha_8}$

$e = \sqrt{2+u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{2-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6}$
$+2X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{4-u}X_{-\alpha_1-\alpha_3}$
$\sqrt{2}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{2}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{u}X_{-\alpha_1}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_2} + H_{\alpha_6}) \oplus \mathbb{C}(H_{\alpha_2} + H_{\alpha_8})$

---

36. 00200002 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1$

$x = 6H_{\alpha_1} + 12H_{\alpha_2} + 16H_{\alpha_3} + 24H_{\alpha_4} + 20H_{\alpha_5} + 16H_{\alpha_6} + 12H_{\alpha_7} + 6H_{\alpha_8}$

$e = X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{3}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5})$
$+\sqrt{2}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6})$
$+2X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(2H_{\alpha_1} + 3H_{\alpha_3} + 4H_{\alpha_4} + 4H_{\alpha_5} + 3H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$

---

45. 00400000 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq 2A_1$

$x = 8H_{\alpha_1} + 14H_{\alpha_2} + 18H_{\alpha_3} + 28H_{\alpha_4} + 24H_{\alpha_5} + 20H_{\alpha_6} + 16H_{\alpha_7} + 8H_{\alpha_8}$

$e = 2(X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7})$
$+\sqrt{2}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6})$
$+\sqrt{6}(X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_2} + 2H_{\alpha_3} + 5H_{\alpha_4} + 2H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$

---

46. 02000200 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_2$

$x = 8H_{\alpha_1} + 16H_{\alpha_2} + 20H_{\alpha_3} + 32H_{\alpha_4} + 26H_{\alpha_5} + 20H_{\alpha_6} + 14H_{\alpha_7} + 8H_{\alpha_8}$

$e = 2(X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$+\sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5})$
$+\sqrt{6}(X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 2H_{\alpha_4} - H_{\alpha_6}) \oplus$
$\mathbb{C}(2H_{\alpha_1} + 3H_{\alpha_2} + 5H_{\alpha_3} + 4H_{\alpha_4} + H_{\alpha_5} + H_{\alpha_7})$

---

51. 40000040 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_2$

$x = 12H_{\alpha_1} + 22H_{\alpha_2} + 26H_{\alpha_3} + 40H_{\alpha_4} + 32H_{\alpha_5} + 24H_{\alpha_6} + 16H_{\alpha_7} + 8H_{\alpha_8}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}) + \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}) + \sqrt{10}X_{-\alpha_1}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} - H_{\alpha_7}) \oplus \mathbb{C}(H_{\alpha_6} + 2H_{\alpha_7} + H_{\alpha_8})$

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52. 00200022 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1 + T_1$

$x = 8H_{\alpha_1} + 18H_{\alpha_2} + 22H_{\alpha_3} + 34H_{\alpha_4} + 28H_{\alpha_5} + 22H_{\alpha_6} + 16H_{\alpha_7} + 8H_{\alpha_8}$

$e = \sqrt{6}(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7} + X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}) + \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5-\alpha_6}) + \sqrt{10}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} - H_{\alpha_6}) \oplus \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 5H_{\alpha_4} + 3H_{\alpha_5} + H_{\alpha_7} + H_{\alpha_8})$

---

53. 20002020 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq A_1$

$x = 10H_{\alpha_1} + 20H_{\alpha_2} + 24H_{\alpha_3} + 38H_{\alpha_4} + 32H_{\alpha_5} + 24H_{\alpha_6} + 16H_{\alpha_7} + 8H_{\alpha_8}$

$e = \sqrt{7-u}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + \sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{7}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}) + X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$

$+X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4}+\sqrt{12-u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5}$
$+\sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}+\sqrt{u-2}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$

$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_3}+H_{\alpha_4}+H_{\alpha_7}+H_{\alpha_8})$

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54. 02020020 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq 2A_1$

$x=10H_{\alpha_1}+20H_{\alpha_2}+24H_{\alpha_3}+38H_{\alpha_4}+32H_{\alpha_5}+26H_{\alpha_6}+18H_{\alpha_7}+10H_{\alpha_8}$

$e=3X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}+\sqrt{5}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$
$+X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}+\sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+2\alpha_6+\alpha_7}$
$+\sqrt{5}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}+\sqrt{8}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$

$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_3}+H_{\alpha_4})\oplus\mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+H_{\alpha_5}+H_{\alpha_6}+H_{\alpha_7})$

---

55. 00020200 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq 2A_1$

$x=8H_{\alpha_1}+18H_{\alpha_2}+22H_{\alpha_3}+36H_{\alpha_4}+30H_{\alpha_5}+24H_{\alpha_6}+16H_{\alpha_7}+8H_{\alpha_8}$

$e=\sqrt{5}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}+\sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}+\sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{5}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}+3X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+H_{\alpha_3}+2H_{\alpha_4}+H_{\alpha_6}+H_{\alpha_7})\oplus\mathbb{C}H_{\alpha_5}$

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56. 02020040 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq 3A_1$

$x=12H_{\alpha_1}+26H_{\alpha_2}+30H_{\alpha_3}+48H_{\alpha_4}+40H_{\alpha_5}+32H_{\alpha_6}+22H_{\alpha_7}+12H_{\alpha_8}$

$e=\sqrt{8}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}+\sqrt{18}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}+\sqrt{10}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5}$
$\sqrt{14}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_7}\oplus\mathbb{C}(H_{\alpha_4}+H_{\alpha_5})\oplus\mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+H_{\alpha_3}+H_{\alpha_6})$

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70. 40040000 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq 2A_1$

$x=16H_{\alpha_1}+26H_{\alpha_2}+34H_{\alpha_3}+52H_{\alpha_4}+44H_{\alpha_5}+36H_{\alpha_6}+24H_{\alpha_7}+12H_{\alpha_8}$

$e=\sqrt{12}X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6}+\sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}+\sqrt{12}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{10}(X_{-\alpha_1}+X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5})$

$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}H_{\alpha_5}\oplus\mathbb{C}(H_{\alpha_2}+H_{\alpha_4}-H_{\alpha_7}-H_{\alpha_8})$

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71. 02020200 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}}\simeq T_2$

$x=12H_{\alpha_1}+24H_{\alpha_2}+30H_{\alpha_3}+48H_{\alpha_4}+40H_{\alpha_5}+32H_{\alpha_6}+22H_{\alpha_7}+12H_{\alpha_8}$

$e=\sqrt{12}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}+\sqrt{6}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{6}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}+\sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+\sqrt{10}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5}+\sqrt{12}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$

$\mathfrak{t}^1_{\mathbb{C}}=\mathbb{C}(H_{\alpha_2}-H_{\alpha_7})\oplus\mathbb{C}(H_{\alpha_1}+H_{\alpha_2}+H_{\alpha_3}+2H_{\alpha_4}+H_{\alpha_5}+H_{\alpha_6}+H_{\alpha_7})$

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77. 04020200 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1 + T_1$

$x = 16H_{\alpha_1} + 30H_{\alpha_2} + 38H_{\alpha_3} + 60H_{\alpha_4} + 50H_{\alpha_5} + 40H_{\alpha_6} + 28H_{\alpha_7} + 16H_{\alpha_8}$

$e = 4X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{22-u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{12}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{14-u}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$+\sqrt{12}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5} + \sqrt{8+u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$+\sqrt{u}X_{\alpha_1+2\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6}) \oplus \mathbb{C}H_{\alpha_5}$

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78. 02020220 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_1$

$x = 14H_{\alpha_1} + 30H_{\alpha_2} + 36H_{\alpha_3} + 58H_{\alpha_4} + 48H_{\alpha_5} + 38H_{\alpha_6} + 26H_{\alpha_7} + 14H_{\alpha_8}$

$e = \sqrt{7}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6} + \sqrt{15}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{15}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{12}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+4X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{7}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + \sqrt{12}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + 2H_{\alpha_3} + 2H_{\alpha_4} + 2H_{\alpha_5} + H_{\alpha_6} + H_{\alpha_7})$

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79. 020020220 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 12H_{\alpha_1} + 26H_{\alpha_2} + 32H_{\alpha_3} + 50H_{\alpha_4} + 42H_{\alpha_5} + 32H_{\alpha_6} + 22H_{\alpha_7} + 12H_{\alpha_8}$

$e = \sqrt{2}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{18}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7}$
$+\sqrt{10}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5} + \sqrt{2}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6}$
$+\sqrt{14}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 3H_{\alpha_4} + 3H_{\alpha_5} + 2H_{\alpha_6} + 3H_{\alpha_7})$

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80. 00400040 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 12H_{\alpha_1} + 26H_{\alpha_2} + 30H_{\alpha_3} + 48H_{\alpha_4} + 40H_{\alpha_5} + 32H_{\alpha_6} + 24H_{\alpha_7} + 12H_{\alpha_8}$

$e = \sqrt{2}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7} + \sqrt{8}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{18}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{2}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6} + \sqrt{10}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{14}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_2} + 5H_{\alpha_3} + 5H_{\alpha_4} + 5H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7})$

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82. 04020240 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = 20H_{\alpha_1} + 42H_{\alpha_2} + 50H_{\alpha_3} + 80H_{\alpha_4} + 66H_{\alpha_5} + 52H_{\alpha_6} + 36H_{\alpha_7} + 20H_{\alpha_8}$

$e = \sqrt{30}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{42}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+4(X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5})$
$+\sqrt{30}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + \sqrt{22}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}(H_{\alpha_1} + H_{\alpha_2} + H_{\alpha_3} + 2H_{\alpha_4} + H_{\alpha_6}) \oplus \mathbb{C}H_{\alpha_5}$

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93. 02022022 $\mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq T_1$

$x = 16H_{\alpha_1} + 34H_{\alpha_2} + 42H_{\alpha_3} + 66H_{\alpha_4} + 56H_{\alpha_5} + 44H_{\alpha_6} + 30H_{\alpha_7} + 16H_{\alpha_8}$

$e = \sqrt{2-u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{24}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{28}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7} + \sqrt{14+u}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-\alpha_5-\alpha_6}$
$+\sqrt{16-u}X_{-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7} + \sqrt{18}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6}$

$\mathfrak{k}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} + H_{\alpha_7})$

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94. 40040040 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq T_1$

$x = 20H_{\alpha_1} + 38H_{\alpha_2} + 46H_{\alpha_3} + 72H_{\alpha_4} + 60H_{\alpha_5} + 48H_{\alpha_6} + 32H_{\alpha_7} + 16H_{\alpha_8}$

$e = \sqrt{30+u}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+\alpha_6} + \sqrt{24}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7}$
$+\sqrt{10}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+\alpha_6+\alpha_7} + \sqrt{18}X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7}$
$+\sqrt{-14-u}X_{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6} + \sqrt{28}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5}$
$+\sqrt{16+u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + \sqrt{u}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+2\alpha_5+\alpha_6+\alpha_7+\alpha_8}$

$\mathfrak{k}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_4} - H_{\alpha_7} - H_{\alpha_8})$

EIX.

Let $\Delta = \{\alpha_1, \alpha_2, \ldots, \alpha_8\}$ be the Bourbaki simple roots of $\mathfrak{g}_{\mathbb{C}}$ then $\Delta_k = \{\beta_1, \ldots, \beta_8\}$, where $\beta_1 = \alpha_1$, $\beta_2 = \alpha_2$, $\beta_3 = \alpha_3$, $\beta_4 = \alpha_4$, $\beta_5 = \alpha_5$, $\beta_6 = \alpha_6$, $\beta_7 = \alpha_7$ and $\beta_8 = 2\alpha_1 + 3\alpha_2 + 4\alpha_3 + 6\alpha_4 + 5\alpha_5 + 4\alpha_6 + 3\alpha_7 + 2\alpha_8$, is a set of simple roots for $\mathfrak{k}_{\mathbb{C}} = E_7 \oplus \mathfrak{sl}_2(\mathbb{C})$.

Table XII

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6. 0000000 4 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq E_6$

$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 8H_{\alpha_6} + 6H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{2}(X_{\alpha_8} + X_{2\alpha_1+3\alpha_2+4\alpha_3+6\alpha_4+5\alpha_5+4\alpha_6+3\alpha_7+\alpha_8})$

$\mathfrak{k}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_1} \oplus \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6}$

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7. 0000002 2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq F_4$

$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 8H_{\alpha_6} + 6H_{\alpha_7} + 2H_{\alpha_8}$

$e = X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_8}$
$+X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{2\alpha_1+3\alpha_2+4\alpha_3+6\alpha_4+5\alpha_5+4\alpha_6+2\alpha_7+\alpha_8}$

$\mathfrak{k}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5}$

---

8. 2000000 0 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq E_6$

$x = 4H_{\alpha_1} + 4H_{\alpha_2} + 6H_{\alpha_3} + 8H_{\alpha_4} + 6H_{\alpha_5} + 4H_{\alpha_6} + 2H_{\alpha_7}$

$e = \sqrt{2}(X_{-\alpha_8} + X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}(H_{\alpha_1} + 2H_{\alpha_7} + H_{\alpha_8}) \oplus \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6}$

---

14. 0000020 0 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq G_2 + 2A_1$

$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 8H_{\alpha_6} + 4H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_6-\alpha_7-\alpha_8}$
$+X_{\alpha_1+3\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8})$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_7} \oplus \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_8})$

---

18. 0000020 4 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq D_4$

$x = 8H_{\alpha_1} + 12H_{\alpha_2} + 16H_{\alpha_3} + 24H_{\alpha_4} + 20H_{\alpha_5} + 16H_{\alpha_6} + 10H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{4-u}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + \sqrt{2-u}X_{-\alpha_8}$
$2X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{u}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{2+u}X_{-\alpha_7-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_2}$

---

19. 0000004 0 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq D_4$

$x = 4H_{\alpha_1} + 6H_{\alpha_2} + 8H_{\alpha_3} + 12H_{\alpha_4} + 10H_{\alpha_5} + 8H_{\alpha_6} + 6H_{\alpha_7}$

$e = 2X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{2+u}X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8}$
$\sqrt{2-u}X_{2\alpha_1+3\alpha_2+4\alpha_3+6\alpha_4+5\alpha_5+4\alpha_6+2\alpha_7+\alpha_8} + \sqrt{u}X_{2\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5 2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{4-u}X_{-\alpha_7-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_2}$

---

20. 2000002 2 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq B_3$

$x = 8H_{\alpha_1} + 10H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 8H_{\alpha_7} + 2H_{\alpha_8}$

$e = X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$\sqrt{3}(X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8})$
$+2X_{-\alpha_2-\alpha_3-2\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_4} + H_{\alpha_5}) \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_6})$

---

21. 0000004 8 $\quad \mathfrak{k}^{(x,e,f)}_{\mathbb{C}} \simeq F_4$

$x = 12H_{\alpha_1} + 18H_{\alpha_2} + 24H_{\alpha_3} + 36H_{\alpha_4} + 30H_{\alpha_5} + 24H_{\alpha_6} + 18H_{\alpha_7} + 8H_{\alpha_8}$

$e = \sqrt{6}(X_{\alpha_7+\alpha_8} + X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8})$
$+\sqrt{10}X_{-\alpha_8}$

$\mathfrak{t}^1_{\mathbb{C}} = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5}$

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22. 2000004 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B4$

$x = 12H_{\alpha_1} + 16H_{\alpha_2} + 22H_{\alpha_3} + 32H_{\alpha_4} + 26H_{\alpha_5} + 20H_{\alpha_6} + 14H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{6}(X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_7-\alpha_8}$
$+X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8})$
$+\sqrt{10}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5}$

---

24. 4000000 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3$

$x = 12H_{\alpha_1} + 14H_{\alpha_2} + 20H_{\alpha_3} + 28H_{\alpha_4} + 22H_{\alpha_5} + 16H_{\alpha_6} + 10H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{6}(X_{\alpha_1++\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8} + X_{\alpha_1+3\alpha_2+3\alpha_3+5\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8})$
$+2(X_{-\alpha_8} + X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-2\alpha_7-\alpha_8})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_3} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6}$

---

25. 2000020 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_3 + T_1$

$x = 8H_{\alpha_1} + 10H_{\alpha_2} + 14H_{\alpha_3} + 20H_{\alpha_4} + 16H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$

$e = \sqrt{6}(X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8})$
$+2(X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_3}+H_{\alpha_4}) \oplus \mathbb{C}(H_{\alpha_1}+H_{\alpha_3}+2H_{\alpha_6}+2H_{\alpha_7}+H_{\alpha_8}) \oplus \mathbb{C}(H_{\alpha_4}+H_{\alpha_5})$

---

28. 0002000 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = 8H_{\alpha_1} + 12H_{\alpha_2} + 16H_{\alpha_3} + 24H_{\alpha_4} + 18H_{\alpha_5} + 12H_{\alpha_6} + 6H_{\alpha_7}$

$e = \sqrt{2}(X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8})$
$+2(X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-2\alpha_7-\alpha_8})$
$+\sqrt{6}(X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8} + X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+4\alpha_5+3\alpha_6+2\alpha_7+\alpha_8})$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_1} + H_{\alpha_3} + 3H_{\alpha_4} + H_{\alpha_5} + 2H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$

---

30. 2000022 2 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2$

$x = 12H_{\alpha_1} + 16H_{\alpha_2} + 22H_{\alpha_3} + 32H_{\alpha_4} + 26H_{\alpha_5} + 20H_{\alpha_6} + 12H_{\alpha_7} + 2H_{\alpha_8}$

$e = \sqrt{5}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{8}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{5}X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{8}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+3X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4} + H_{\alpha_5})$

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31. 0000040 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2$

$x = 12H_{\alpha_1} + 18H_{\alpha_2} + 24H_{\alpha_3} + 36H_{\alpha_4} + 30H_{\alpha_5} + 24H_{\alpha_6} + 14H_{\alpha_7} + 4H_{\alpha_8}$

$e = X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + \sqrt{5}X_{\alpha_1+\alpha_2+2\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{8}(X_{-\alpha_1-2\alpha_2-2\alpha_3-3\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8} + X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8})$
$+\sqrt{5}X_{\alpha_1+2\alpha_2+2\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + 3X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_4})$

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32. 2000024 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_3$

$x = 16H_{\alpha_1} + 22H_{\alpha_2} + 30H_{\alpha_3} + 44H_{\alpha_4} + 36H_{\alpha_5} + 28H_{\alpha_6} + 18H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{14}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{18}X_{2\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{10}(X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8} + X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8})$
$+\sqrt{8}X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}(H_{\alpha_3} + H_{\alpha_5})$

---

33. 4000004 8 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq B_3$

$x = 20H_{\alpha_1} + 26H_{\alpha_2} + 36H_{\alpha_3} + 52H_{\alpha_4} + 42H_{\alpha_5} + 32H_{\alpha_6} + 22H_{\alpha_7} + 8H_{\alpha_8}$

$e = \sqrt{14}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{10}X_{\alpha_1+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{10}X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + \sqrt{18}X_{-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{8}X_{-\alpha_2-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}H_{\alpha_6}$

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34. 0002020 0 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq 2A_1$

$x = 12H_{\alpha_1} + 18H_{\alpha_2} + 24H_{\alpha_3} + 36H_{\alpha_4} + 28H_{\alpha_5} + 20H_{\alpha_6} + 10H_{\alpha_7}$

$e = \sqrt{6}X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{10}X_{\alpha_1+\alpha_2+2\alpha_3+3\alpha_4+3\alpha_5+3\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{12}X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + \sqrt{12}X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{10}X_{-\alpha_1-\alpha_2-2\alpha_3-3\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8} + \sqrt{6}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-2\alpha_7-\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_5} \oplus \mathbb{C}(2H_{\alpha_1} + 2H_{\alpha_2} + 3H_{\alpha_3} + 3H_{\alpha_4} + 2H_{\alpha_6} + H_{\alpha_7} + H_{\alpha_8})$

---

35. 4000040 4 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq A_2$

$x = 20H_{\alpha_1} + 26H_{\alpha_2} + 36H_{\alpha_3} + 52H_{\alpha_4} + 42H_{\alpha_5} + 32H_{\alpha_6} + 18H_{\alpha_7} + 4H_{\alpha_8}$

$e = \sqrt{22}X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+2\alpha_7+\alpha_8} + \sqrt{u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{22-u}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8}$
$+\sqrt{12}X_{-\alpha_1-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8} + \sqrt{12}X_{-\alpha_1-\alpha_2-2\alpha_3-2\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{8-u}X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8} + \sqrt{8+u}X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-2\alpha_7-\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_2} \oplus \mathbb{C}H_{\alpha_4}$

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36. 4000044 8 $\quad \mathfrak{k}_{\mathbb{C}}^{(x,e,f)} \simeq G_2$

$x = 28H_{\alpha_1} + 38H_{\alpha_2} + 52H_{\alpha_3} + 76H_{\alpha_4} + 62H_{\alpha_5} + 48H_{\alpha_6} + 30H_{\alpha_7} + 8H_{\alpha_8}$

$e = \sqrt{30}X_{\alpha_1+\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + \sqrt{22}X_{\alpha_2+\alpha_3+2\alpha_4+2\alpha_5+2\alpha_6+2\alpha_7+\alpha_8}$
$+\sqrt{30}X_{\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4+3\alpha_5+2\alpha_6+\alpha_7+\alpha_8} + 4X_{-\alpha_1-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8}$
$+\sqrt{42}X_{-\alpha_2-\alpha_3-2\alpha_4-2\alpha_5-2\alpha_6-\alpha_7-\alpha_8} + 4X_{-\alpha_1-2\alpha_2-2\alpha_3-3\alpha_4-2\alpha_5-\alpha_6-\alpha_7-\alpha_8}$

$\mathfrak{k}_{\mathbb{C}}^1 = \mathbb{C}H_{\alpha_4} \oplus \mathbb{C}H_{\alpha_5}$

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