PREHOMOGENEOUS SPACES ASSOCIATED WITH NILPOTENT ORBITS IN $E_7(\mathbb{C})$

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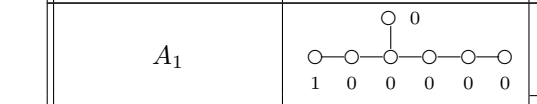
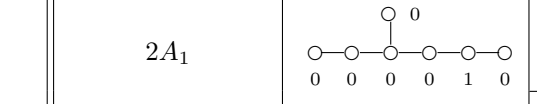
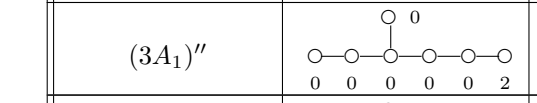
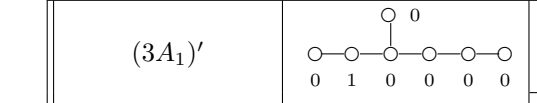
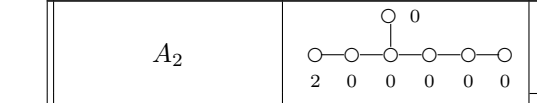
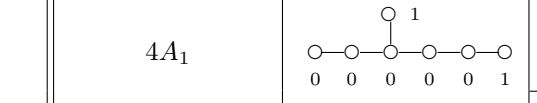
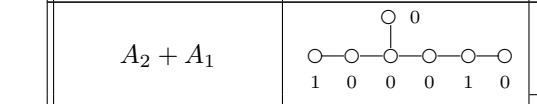
Nilpotent orbits in type E_7				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
A_1		1	32	$(-1,0,1,0,0,0,0)$
		2	1	$(1,0,0,0,0,0,0)$
$2A_1$		1	32	$(0,1,0,0,0,-1,1)$
		2	10	$(1,0,0,0,0,0,0)$
$(3A_1)''$		2	27	$(1,0,0,0,0,0,0)$
$(3A_1)'$		1	30	$(1,0,-1,0,0,1,0)$
		2	15	$(0,0,-1,1,0,0,0)$
		3	2	$(1,0,0,0,0,0,0)$
A_2		2	32	$(-1,0,1,0,0,0,0)$
		4	1	$(1,0,0,0,0,0,0)$
$4A_1$		1	26	$(1,-1,0,0,0,0,1)$ $(0,-1,0,1,0,0,-1)$
		2	16	$(0,1,0,0,0,0,-1)$ $(0,-1,0,0,1,0,0)$
		3	6	$(1,0,0,0,0,0,0)$
A_2+A_1		1	24	$(0,0,1,0,0,-1,0)$ $(-1,0,0,0,1,-1,1)$
		2	17	$(-1,0,0,0,0,1,0)$ $(0,1,0,0,0,-1,1)$
		3	8	$(-1,0,1,0,0,0,0)$
		4	1	$(1,0,0,0,0,0,0)$

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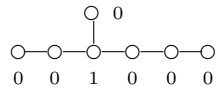
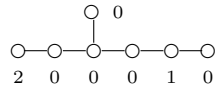
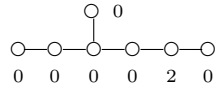
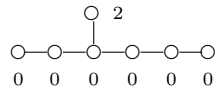
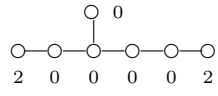
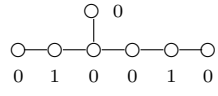
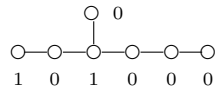
Key words and phrases. Lie group, nilpotent orbit, prehomogeneous space.

Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
$A_2 + 2A_1$		1	24	$(1,1,0,-1,0,0,1)$
		2	18	$(0,0,1,-1,0,1,0)$
		3	8	$(0,1,0,-1,1,0,0)$
		4	3	$(1,0,0,0,0,0,0)$
A_3		1	16	$(-1,0,0,0,1,-1,1)$
		2	9	$(0,0,1,0,0,-1,0)$ $(-1,0,0,0,0,1,0)$
		3	16	$(0,1,0,0,0,-1,1)$
		4	8	$(-1,0,1,0,0,0,0)$
		6	1	$(1,0,0,0,0,0,0)$
$2A_2$		2	32	$(0,1,0,0,0,-1,1)$
		4	10	$(1,0,0,0,0,0,0)$
$A_2 + 3A_1$		2	35	$(0,-1,0,0,1,0,0)$
		4	7	$(1,0,0,0,0,0,0)$
$(A_3 + A_1)''$		2	26	$(-1,0,0,0,0,1,0)$ $(0,1,0,0,0,0,-1)$
		4	16	$(-1,0,1,0,0,0,0)$
		6	1	$(1,0,0,0,0,0,0)$
$2A_2 + A_1$		1	20	$(0,1,-1,0,0,0,1)$ $(1,0,-1,1,0,-1,0)$
		2	17	$(0,0,1,0,0,-1,0)$ $(1,0,-1,0,1,-1,1)$
		3	10	$(1,0,-1,0,0,1,0)$ $(0,1,0,0,0,-1,1)$
		4	6	$(0,0,-1,1,0,0,0)$
		5	2	$(1,0,0,0,0,0,0)$
$(A_3 + A_1)'$		1	18	$(1,0,1,-1,0,0,0)$ $(-1,1,1,-1,0,0,1)$
		2	14	$(1,1,0,-1,0,0,1)$ $(-1,0,0,0,0,1,0)$
		3	12	$(0,0,1,-1,0,1,0)$
		4	8	$(0,1,0,-1,1,0,0)$
		5	2	$(-1,0,1,0,0,0,0)$
		6	1	$(1,0,0,0,0,0,0)$

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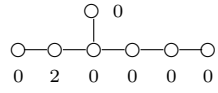
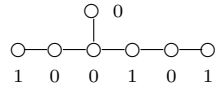
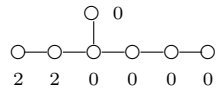
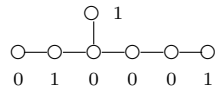
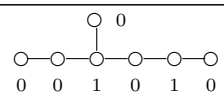
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
$D_4(a_1)$		2	30	$(1,0,-1,0,0,1,0)$
		4	15	$(0,0,-1,1,0,0,0)$
		6	2	$(1,0,0,0,0,0,0)$
A_3+2A_1		1	18	$(0,0,0,0,-1,1,1)$ $(1,1,0,0,-1,0,0)$ $(-1,0,0,1,-1,1,-1)$
		2	15	$(-1,0,0,0,1,0,-1)$ $(-1,0,0,1,-1,0,1)$ $(0,0,1,0,-1,1,-1)$
		3	10	$(0,0,1,0,-1,0,1)$ $(-1,0,0,0,0,1,0)$ $(0,1,0,0,0,0,-1)$
		4	8	$(0,1,0,0,-1,1,0)$
		5	4	$(-1,0,1,0,0,0,0)$
		6	1	$(1,0,0,0,0,0,0)$
D_4		2	16	$(2,0,-1,0,0,0,0)$ $(-1,0,0,0,0,1,0)$
		4	15	$(1,0,-1,0,0,1,0)$
		6	15	$(0,0,-1,1,0,0,0)$
		8	1	$(-1,0,1,0,0,0,0)$
		10	1	$(1,0,0,0,0,0,0)$
$D_4(a_1)+A_1$		1	16	$(0,1,-1,0,0,1,-1)$ $(0,-1,-1,1,0,0,1)$ $(1,-1,0,0,0,1,-1)$
		2	15	$(0,1,-1,0,0,0,1)$ $(1,-1,0,0,0,0,1)$ $(1,0,-1,0,1,0,-1)$
		3	12	$(0,-1,0,1,0,0,-1)$ $(1,0,-1,0,0,1,0)$
		4	7	$(0,1,0,0,0,0,-1)$ $(0,-1,0,0,1,0,0)$
		5	4	$(0,0,-1,1,0,0,0)$
		6	2	$(1,0,0,0,0,0,0)$
A_3+A_2		1	16	$(0,0,0,-1,1,0,1)$ $(1,1,0,-1,1,-1,0)$
		2	15	$(0,0,1,0,0,-1,0)$ $(1,1,0,-1,0,0,1)$
		3	12	$(0,0,1,-1,1,-1,1)$
		4	7	$(0,0,1,-1,0,1,0)$ $(0,1,0,0,0,-1,1)$
		5	4	$(0,1,0,-1,1,0,0)$

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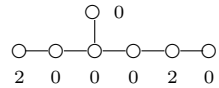
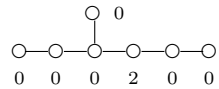
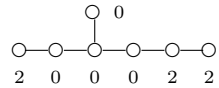
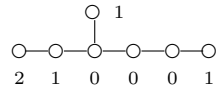
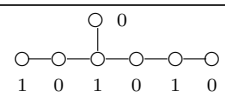
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		6	3	$(1,0,0,0,0,0,0)$
A_4		2	24	$(0,0,1,0,0,-1,0)$ $(-1,0,0,0,1,-1,1)$
		4	17	$(-1,0,0,0,0,1,0)$ $(0,1,0,0,0,-1,1)$
		6	8	$(-1,0,1,0,0,0,0)$
		8	1	$(1,0,0,0,0,0,0)$
$A_3+A_2+A_1$		2	30	$(0,0,1,0,-1,0,1)$
		4	15	$(0,1,0,0,-1,1,0)$
		6	5	$(1,0,0,0,0,0,0)$
$(A_5)''$		2	17	$(0,0,0,0,0,-1,2)$ $(0,0,1,0,0,-1,0)$ $(-1,0,0,0,1,0,-1)$
		4	16	$(-1,0,0,0,1,-1,1)$ $(0,1,0,0,0,0,-1)$
		6	9	$(-1,0,0,0,0,1,0)$ $(0,1,0,0,0,-1,1)$
		8	8	$(-1,0,1,0,0,0,0)$
		10	1	$(1,0,0,0,0,0,0)$
D_4+A_1		1	12	$(0,1,-1,0,0,1,-1)$ $(-1,-1,1,0,0,1,-1)$ $(0,-1,-1,1,0,0,1)$
		2	9	$(2,0,-1,0,0,0,0)$ $(0,1,-1,0,0,0,1)$ $(-1,-1,1,0,0,0,1)$ $(-1,0,0,0,1,0,-1)$
		3	8	$(1,-1,0,0,0,1,-1)$ $(-1,0,0,0,0,1,0)$
		4	7	$(1,-1,0,0,0,0,1)$ $(1,0,-1,0,1,0,-1)$
		5	8	$(0,-1,0,1,0,0,-1)$ $(1,0,-1,0,0,1,0)$
		6	7	$(0,1,0,0,0,0,-1)$ $(0,-1,0,0,1,0,0)$
		7	4	$(0,0,-1,1,0,0,0)$
		8	1	$(-1,0,1,0,0,0,0)$
		10	1	$(1,0,0,0,0,0,0)$
A_4+A_1		1	14	$(1,0,1,-1,0,0,0)$ $(0,0,0,-1,1,0,1)$ $(-1,1,1,-1,1,-1,0)$

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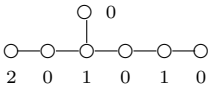
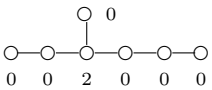
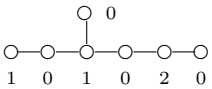
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		2	13	$(1,1,0,-1,1,-1,0)$ $(-1,0,0,1,0,-1,0)$ $(-1,1,1,-1,0,0,1)$
		3	10	$(0,0,1,0,0,-1,0)$ $(1,1,0,-1,0,0,1)$ $(-1,0,0,0,1,-1,1)$
		4	9	$(-1,0,0,0,0,1,0)$ $(0,0,1,-1,1,-1,1)$
		5	6	$(0,0,1,-1,0,1,0)$ $(0,1,0,0,0,-1,1)$
		6	4	$(0,1,0,-1,1,0,0)$
		7	2	$(-1,0,1,0,0,0,0)$
		8	1	$(1,0,0,0,0,0,0)$
$D_5(a_1)$		1	12	$(0,0,0,-1,1,0,1)$ $(-1,1,1,-1,1,-1,0)$
		2	11	$(1,0,1,-1,0,0,0)$ $(-1,0,0,1,0,-1,0)$ $(-1,1,1,-1,0,0,1)$
		3	8	$(1,1,0,-1,1,-1,0)$ $(-1,0,0,0,1,-1,1)$
		4	7	$(0,0,1,0,0,-1,0)$ $(1,1,0,-1,0,0,1)$ $(-1,0,0,0,0,1,0)$
		5	8	$(0,0,1,-1,1,-1,1)$
		6	6	$(0,0,1,-1,0,1,0)$ $(0,1,0,0,0,-1,1)$
		7	4	$(0,1,0,-1,1,0,0)$
		8	2	$(-1,0,1,0,0,0,0)$
		10	1	$(1,0,0,0,0,0,0)$
A_4+A_2		2	24	$(1,1,0,-1,0,0,1)$
		4	18	$(0,0,1,-1,0,1,0)$
		6	8	$(0,1,0,-1,1,0,0)$
		8	3	$(1,0,0,0,0,0,0)$
$(A_5)'$		1	10	$(1,0,1,-1,0,0,0)$ $(-1,1,1,-1,1,-1,0)$
		2	9	$(0,0,0,-1,1,0,1)$ $(1,1,0,-1,1,-1,0)$ $(-1,0,0,1,0,-1,0)$
		3	10	$(-1,1,1,-1,0,0,1)$ $(0,0,1,0,0,-1,0)$
		4	8	$(1,1,0,-1,0,0,1)$ $(-1,0,0,0,1,-1,1)$

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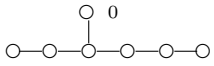
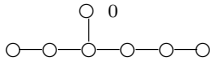
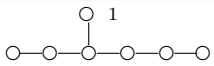
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		5	8	$(0,0,1,-1,1,-1,1)$
		6	5	$(-1,0,0,0,0,1,0)$ $(0,1,0,0,0,-1,1)$
		7	2	$(0,0,1,-1,0,1,0)$
		8	4	$(0,1,0,-1,1,0,0)$
		9	2	$(-1,0,1,0,0,0,0)$
		10	1	$(1,0,0,0,0,0,0)$
A_5+A_1		1	12	$(1,0,1,-1,0,0,0)$ $(0,0,0,-1,1,1,-1)$ $(-1,1,1,-1,1,-1,0)$
		2	10	$(0,0,0,0,0,-1,2)$ $(1,1,0,-1,1,-1,0)$ $(-1,0,0,1,0,-1,0)$ $(-1,1,1,-1,0,1,-1)$
		3	8	$(0,0,0,-1,1,0,1)$ $(1,1,0,-1,0,1,-1)$ $(0,0,1,0,0,-1,0)$ $(-1,0,0,0,1,0,-1)$
		4	8	$(-1,1,1,-1,0,0,1)$ $(0,0,1,-1,1,0,-1)$
		5	6	$(1,1,0,-1,0,0,1)$ $(-1,0,0,0,1,-1,1)$ $(0,1,0,0,0,0,-1)$
		6	5	$(-1,0,0,0,0,1,0)$ $(0,0,1,-1,1,-1,1)$
		7	4	$(0,0,1,-1,0,1,0)$ $(0,1,0,0,0,-1,1)$
		8	4	$(0,1,0,-1,1,0,0)$
		9	2	$(-1,0,1,0,0,0,0)$
		10	1	$(1,0,0,0,0,0,0)$
$D_5(a_1)+A_1$		2	22	$(1,1,0,0,-1,0,0)$ $(-1,0,0,1,-1,0,1)$
		4	15	$(0,0,1,0,-1,0,1)$ $(-1,0,0,0,0,1,0)$
		6	12	$(0,1,0,0,-1,1,0)$
		8	4	$(-1,0,1,0,0,0,0)$
		10	1	$(1,0,0,0,0,0,0)$
$D_6(a_2)$		1	10	$(0,1,-1,1,-1,0,0)$ $(1,-1,0,1,-1,0,0)$ $(0,-1,-1,1,0,1,-1)$
		2	10	$(0,0,0,0,-1,1,1)$ $(1,1,0,0,-1,0,0)$ $(0,1,-1,0,0,1,-1)$ $(1,-1,0,0,0,1,-1)$

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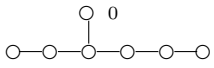
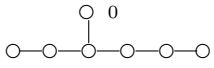
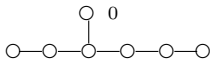
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		3	10	$(0,-1,-1,1,0,0,1)$ $(1,0,-1,1,-1,1,-1)$
		4	7	$(0,1,-1,0,0,0,1)$ $(1,-1,0,0,0,0,1)$ $(0,0,1,0,-1,1,-1)$ $(1,0,-1,0,1,0,-1)$
		5	6	$(1,0,-1,1,-1,0,1)$ $(0,-1,0,1,0,0,-1)$
		6	6	$(0,0,1,0,-1,0,1)$ $(1,0,-1,0,0,1,0)$ $(0,1,0,0,0,0,-1)$
		7	4	$(0,-1,0,1,-1,1,0)$
		8	3	$(0,1,0,0,-1,1,0)$ $(0,-1,0,0,1,0,0)$
		9	2	$(0,0,-1,1,0,0,0)$
		10	2	$(1,0,0,0,0,0,0)$
$E_6(a_3)$		2	20	$(0,1,-1,0,0,0,1)$ $(1,0,-1,1,0,-1,0)$
		4	17	$(0,0,1,0,0,-1,0)$ $(1,0,-1,0,1,-1,1)$
		6	10	$(1,0,-1,0,0,1,0)$ $(0,1,0,0,0,-1,1)$
		8	6	$(0,0,-1,1,0,0,0)$
		10	2	$(1,0,0,0,0,0,0)$
D_5		2	15	$(2,0,-1,0,0,0,0)$ $(-1,0,0,1,0,-1,0)$ $(0,1,-1,0,0,0,1)$
		4	14	$(1,0,-1,1,0,-1,0)$ $(-1,0,0,0,1,-1,1)$
		6	10	$(0,0,1,0,0,-1,0)$ $(1,0,-1,0,1,-1,1)$ $(-1,0,0,0,0,1,0)$
		8	9	$(1,0,-1,0,0,1,0)$ $(0,1,0,0,0,-1,1)$
		10	6	$(0,0,-1,1,0,0,0)$
		12	1	$(-1,0,1,0,0,0,0)$
		14	1	$(1,0,0,0,0,0,0)$
$E_7(a_5)$		2	21	$(0,0,0,-1,1,0,1)$ $(1,1,0,-1,0,1,-1)$
		4	15	$(1,1,0,-1,0,0,1)$ $(0,0,1,-1,1,0,-1)$
		6	11	$(0,1,0,0,0,0,-1)$ $(0,0,1,-1,0,1,0)$

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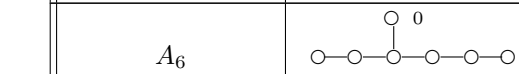
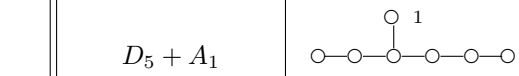
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		8	6	$(0,1,0,-1,1,0,0)$
		10	3	$(1,0,0,0,0,0,0)$
A_6		2	16	$(0,0,0,-1,1,0,1)$ $(1,1,0,-1,1,-1,0)$
		4	15	$(0,0,1,0,0,-1,0)$ $(1,1,0,-1,0,0,1)$
		6	12	$(0,0,1,-1,1,-1,1)$
		8	7	$(0,0,1,-1,0,1,0)$ $(0,1,0,0,0,-1,1)$
		10	4	$(0,1,0,-1,1,0,0)$
		12	3	$(1,0,0,0,0,0,0)$
D_5+A_1		1	8	$(0,1,-1,1,-1,0,0)$ $(-1,-1,1,1,-1,0,0)$ $(0,-1,-1,1,1,-1,0)$ $(0,0,0,0,-1,1,1)$
		2	8	$(2,0,-1,0,0,0,0)$ $(-1,1,1,0,-1,0,0)$ $(0,1,-1,0,1,-1,0)$ $(-1,-1,1,0,1,-1,0)$ $(0,-1,-1,1,0,0,1)$
		3	8	$(1,-1,0,1,-1,0,0)$ $(-1,0,0,1,0,-1,0)$ $(0,1,-1,0,0,0,1)$ $(-1,-1,1,0,0,0,1)$
		4	6	$(1,1,0,0,-1,0,0)$ $(1,-1,0,0,1,-1,0)$ $(-1,0,0,1,-1,0,1)$
		5	6	$(1,0,-1,1,0,-1,0)$ $(1,-1,0,0,0,0,1)$ $(-1,0,0,0,1,-1,1)$
		6	6	$(0,0,1,0,0,-1,0)$ $(1,0,-1,1,-1,0,1)$ $(-1,0,0,0,0,1,0)$
		7	4	$(0,0,1,0,-1,0,1)$ $(1,0,-1,0,1,-1,1)$
		8	5	$(1,0,-1,0,0,1,0)$ $(0,-1,0,1,0,-1,1)$
		9	4	$(0,1,0,0,0,-1,1)$ $(0,-1,0,1,-1,1,0)$
		10	2	$(0,1,0,0,-1,1,0)$ $(0,-1,0,0,1,0,0)$
		11	2	$(0,0,-1,1,0,0,0)$
		12	1	$(-1,0,1,0,0,0,0)$
		14	1	$(1,0,0,0,0,0,0)$

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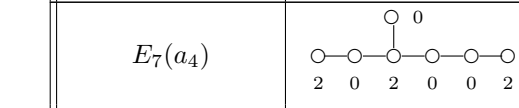
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
$D_6(a_1)$		1	8	$(0,1,-1,1,-1,0,0)$ $(-1,-1,1,1,-1,0,0)$ $(0,-1,-1,1,0,1,-1)$
		2	8	$(2,0,-1,0,0,0,0)$ $(0,0,0,0,-1,1,1)$ $(-1,1,1,0,-1,0,0)$ $(0,1,-1,0,0,1,-1)$ $(-1,-1,1,0,0,1,-1)$
		3	8	$(1,-1,0,1,-1,0,0)$ $(0,-1,-1,1,0,0,1)$ $(-1,0,0,1,-1,1,-1)$
		4	6	$(1,1,0,0,-1,0,0)$ $(1,-1,0,0,0,1,-1)$ $(0,1,-1,0,0,0,1)$ $(-1,-1,1,0,0,0,1)$ $(-1,0,0,0,1,0,-1)$
		5	6	$(1,0,-1,1,-1,1,-1)$ $(-1,0,0,1,-1,0,1)$
		6	6	$(1,-1,0,0,0,0,1)$ $(0,0,1,0,-1,1,-1)$ $(1,0,-1,0,1,0,-1)$ $(-1,0,0,0,0,1,0)$
		7	4	$(1,0,-1,1,-1,0,1)$ $(0,-1,0,1,0,0,-1)$
		8	4	$(0,0,1,0,-1,0,1)$ $(1,0,-1,0,0,1,0)$ $(0,1,0,0,0,0,-1)$
		9	4	$(0,-1,0,1,-1,1,0)$
		10	3	$(0,1,0,0,-1,1,0)$ $(0,-1,0,0,1,0,0)$
		11	2	$(0,0,-1,1,0,0,0)$
		12	1	$(-1,0,1,0,0,0,0)$
		14	1	$(1,0,0,0,0,0,0)$
$E_7(a_4)$		2	17	$(1,0,1,-1,0,0,0)$ $(0,0,0,-1,1,0,1)$ $(-1,1,1,-1,0,1,-1)$
		4	13	$(1,1,0,-1,0,1,-1)$ $(-1,1,1,-1,0,0,1)$ $(-1,0,0,0,1,0,-1)$
		6	11	$(1,1,0,-1,0,0,1)$ $(0,0,1,-1,1,0,-1)$ $(-1,0,0,0,0,1,0)$
		8	8	$(0,1,0,0,0,0,-1)$ $(0,0,1,-1,0,1,0)$
		10	6	$(0,1,0,-1,1,0,0)$

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Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		12	2	$(-1,0,1,0,0,0,0)$
		14	1	$(1,0,0,0,0,0,0)$
D_6		1	6	$(0,1,-1,1,-1,0,0)$ $(-1,-1,1,1,-1,0,0)$ $(0,-1,-1,1,1,-1,0)$
		2	6	$(2,0,-1,0,0,0,0)$ $(0,0,0,0,-1,2,-1)$ $(0,0,0,0,0,-1,2)$ $(-1,1,1,0,-1,0,0)$ $(0,1,-1,0,1,-1,0)$ $(-1,-1,1,0,1,-1,0)$
		3	6	$(1,-1,0,1,-1,0,0)$ $(0,-1,-1,1,0,1,-1)$ $(-1,0,0,1,0,-1,0)$
		4	5	$(0,0,0,0,-1,1,1)$ $(1,1,0,0,-1,0,0)$ $(1,-1,0,0,1,-1,0)$ $(0,1,-1,0,0,1,-1)$ $(-1,-1,1,0,0,1,-1)$
		5	6	$(0,-1,-1,1,0,0,1)$ $(1,0,-1,1,0,-1,0)$ $(-1,0,0,1,-1,1,-1)$
		6	5	$(1,-1,0,0,0,1,-1)$ $(0,1,-1,0,0,0,1)$ $(-1,-1,1,0,0,0,1)$ $(0,0,1,0,0,-1,0)$ $(-1,0,0,0,1,0,-1)$
		7	4	$(1,0,-1,1,-1,1,-1)$ $(-1,0,0,1,-1,0,1)$
		8	4	$(1,-1,0,0,0,0,1)$ $(0,0,1,0,-1,1,-1)$ $(1,0,-1,0,1,0,-1)$ $(-1,0,0,0,1,-1,1)$
		9	4	$(1,0,-1,1,-1,0,1)$ $(0,-1,0,1,0,0,-1)$
		10	4	$(0,0,1,0,-1,0,1)$ $(1,0,-1,0,1,-1,1)$ $(-1,0,0,0,0,1,0)$ $(0,1,0,0,0,0,-1)$
		11	2	$(0,-1,0,1,0,-1,1)$
		12	2	$(1,0,-1,0,0,1,0)$ $(0,1,0,0,0,-1,1)$
		13	2	$(0,-1,0,1,-1,1,0)$
		14	2	$(0,1,0,0,-1,1,0)$ $(0,-1,0,0,1,0,0)$

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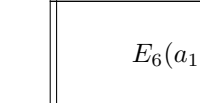
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		15	2	$(0,0,-1,1,0,0,0)$
		16	1	$(-1,0,1,0,0,0,0)$
		18	1	$(1,0,0,0,0,0,0)$
$E_6(a_1)$		2	14	$(1,0,1,-1,0,0,0)$ $(0,0,0,-1,1,0,1)$ $(-1,1,1,-1,1,-1,0)$
		4	13	$(1,1,0,-1,1,-1,0)$ $(-1,0,0,1,0,-1,0)$ $(-1,1,1,-1,0,0,1)$
		6	10	$(0,0,1,0,0,-1,0)$ $(1,1,0,-1,0,0,1)$ $(-1,0,0,0,1,-1,1)$
		8	9	$(-1,0,0,0,0,1,0)$ $(0,0,1,-1,1,-1,1)$
		10	6	$(0,0,1,-1,0,1,0)$ $(0,1,0,0,0,-1,1)$
		12	4	$(0,1,0,-1,1,0,0)$
		14	2	$(-1,0,1,0,0,0,0)$
		16	1	$(1,0,0,0,0,0,0)$
E_6		2	10	$(2,0,-1,0,0,0,0)$ $(-1,0,2,-1,0,0,0)$ $(0,1,-1,0,1,-1,0)$ $(0,0,0,-1,1,0,1)$
		4	9	$(1,0,1,-1,0,0,0)$ $(-1,1,1,-1,1,-1,0)$ $(0,1,-1,0,0,0,1)$
		6	9	$(1,1,0,-1,1,-1,0)$ $(-1,0,0,1,0,-1,0)$ $(-1,1,1,-1,0,0,1)$
		8	9	$(1,0,-1,1,0,-1,0)$ $(1,1,0,-1,0,0,1)$ $(-1,0,0,0,1,-1,1)$
		10	6	$(0,0,1,0,0,-1,0)$ $(1,0,-1,0,1,-1,1)$ $(-1,0,0,0,0,1,0)$
		12	5	$(0,0,1,-1,1,-1,1)$ $(1,0,-1,0,0,1,0)$
		14	5	$(0,0,1,-1,0,1,0)$ $(0,1,0,0,0,-1,1)$
		16	4	$(0,1,0,-1,1,0,0)$
		18	1	$(0,0,-1,1,0,0,0)$
		20	1	$(-1,0,1,0,0,0,0)$
		22	1	$(1,0,0,0,0,0,0)$

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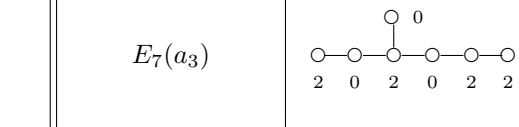
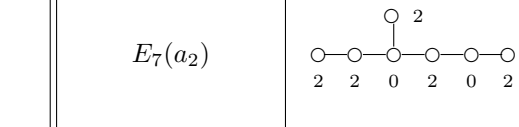
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
$E_7(a_3)$		2	13	$(0,0,0,0,0,-1,2)$ $(1,0,1,-1,0,0,0)$ $(0,0,0,-1,1,1,-1)$ $(-1,1,1,-1,1,-1,0)$
		4	11	$(0,0,0,-1,1,0,1)$ $(1,1,0,-1,1,-1,0)$ $(-1,0,0,1,0,-1,0)$ $(-1,1,1,-1,0,1,-1)$
		6	10	$(1,1,0,-1,0,1,-1)$ $(-1,1,1,-1,0,0,1)$ $(0,0,1,0,0,-1,0)$ $(-1,0,0,0,1,0,-1)$
		8	8	$(1,1,0,-1,0,0,1)$ $(-1,0,0,0,1,-1,1)$ $(0,0,1,-1,1,0,-1)$
		10	7	$(-1,0,0,0,0,1,0)$ $(0,0,1,-1,1,-1,1)$ $(0,1,0,0,0,0,-1)$
		12	4	$(0,0,1,-1,0,1,0)$ $(0,1,0,0,0,-1,1)$
		14	4	$(0,1,0,-1,1,0,0)$
		16	2	$(-1,0,1,0,0,0,0)$
		18	1	$(1,0,0,0,0,0,0)$
$E_7(a_2)$		2	11	$(2,0,-1,0,0,0,0)$ $(0,1,-1,1,-1,0,0)$ $(-1,-1,1,1,-1,0,0)$ $(0,0,0,0,-1,1,1)$ $(0,-1,-1,1,0,1,-1)$
		4	9	$(1,-1,0,1,-1,0,0)$ $(-1,1,1,0,-1,0,0)$ $(0,1,-1,0,0,1,-1)$ $(-1,-1,1,0,0,1,-1)$ $(0,-1,-1,1,0,0,1)$
		6	9	$(1,1,0,0,-1,0,0)$ $(1,-1,0,0,0,1,-1)$ $(0,1,-1,0,0,0,1)$ $(-1,-1,1,0,0,0,1)$ $(-1,0,0,1,-1,1,-1)$
		8	8	$(1,-1,0,0,0,0,1)$ $(1,0,-1,1,-1,1,-1)$ $(-1,0,0,0,1,0,-1)$ $(-1,0,0,1,-1,0,1)$

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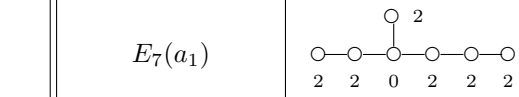
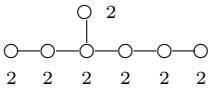
Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		10	7	$(0,0,1,0,-1,1,-1)$ $(1,0,-1,0,1,0,-1)$ $(1,0,-1,1,-1,0,1)$ $(-1,0,0,0,0,1,0)$
		12	5	$(0,0,1,0,-1,0,1)$ $(0,-1,0,1,0,0,-1)$ $(1,0,-1,0,0,1,0)$
		14	5	$(0,1,0,0,0,0,-1)$ $(0,-1,0,1,-1,1,0)$
		16	3	$(0,1,0,0,-1,1,0)$ $(0,-1,0,0,1,0,0)$
		18	2	$(0,0,-1,1,0,0,0)$
		20	1	$(-1,0,1,0,0,0,0)$
		22	1	$(1,0,0,0,0,0,0)$
$E_7(a_1)$		2	9	$(2,0,-1,0,0,0,0)$ $(0,0,0,0,-1,2,-1)$ $(0,0,0,0,0,-1,2)$ $(0,1,-1,1,-1,0,0)$ $(-1,-1,1,1,-1,0,0)$ $(0,-1,-1,1,1,-1,0)$
		4	8	$(0,0,0,0,-1,1,1)$ $(1,-1,0,1,-1,0,0)$ $(-1,1,1,0,-1,0,0)$ $(0,1,-1,0,1,-1,0)$ $(-1,-1,1,0,1,-1,0)$ $(0,-1,-1,1,0,1,-1)$
		6	8	$(1,1,0,0,-1,0,0)$ $(1,-1,0,0,1,-1,0)$ $(0,1,-1,0,0,1,-1)$ $(-1,-1,1,0,0,1,-1)$ $(0,-1,-1,1,0,0,1)$ $(-1,0,0,1,0,-1,0)$
		8	7	$(1,-1,0,0,0,1,-1)$ $(0,1,-1,0,0,0,1)$ $(-1,-1,1,0,0,0,1)$ $(1,0,-1,1,0,-1,0)$ $(-1,0,0,1,-1,1,-1)$
		10	7	$(1,-1,0,0,0,0,1)$ $(0,0,1,0,0,-1,0)$ $(1,0,-1,1,-1,1,-1)$ $(-1,0,0,0,1,0,-1)$ $(-1,0,0,1,-1,0,1)$

(continued on next page)

Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		12	5	$(0,0,1,0,-1,1,-1)$ $(1,0,-1,0,1,0,-1)$ $(1,0,-1,1,-1,0,1)$ $(-1,0,0,0,1,-1,1)$
		14	5	$(0,0,1,0,-1,0,1)$ $(1,0,-1,0,1,-1,1)$ $(-1,0,0,0,0,1,0)$ $(0,-1,0,1,0,0,-1)$
		16	4	$(1,0,-1,0,0,1,0)$ $(0,1,0,0,0,0,-1)$ $(0,-1,0,1,0,-1,1)$
		18	3	$(0,1,0,0,0,-1,1)$ $(0,-1,0,1,-1,1,0)$
		20	2	$(0,1,0,0,-1,1,0)$ $(0,-1,0,0,1,0,0)$
		22	2	$(0,0,-1,1,0,0,0)$
		24	1	$(-1,0,1,0,0,0,0)$
		26	1	$(1,0,0,0,0,0,0)$
E_7		2	7	$(2,0,-1,0,0,0,0)$ $(0,2,0,-1,0,0,0)$ $(-1,0,2,-1,0,0,0)$ $(0,-1,-1,2,-1,0,0)$ $(0,0,0,-1,2,-1,0)$ $(0,0,0,0,-1,2,-1)$ $(0,0,0,0,0,-1,2)$
		4	6	$(1,0,1,-1,0,0,0)$ $(0,1,-1,1,-1,0,0)$ $(-1,-1,1,1,-1,0,0)$ $(0,-1,-1,1,1,-1,0)$ $(0,0,0,-1,1,1,-1)$ $(0,0,0,0,-1,1,1)$
		6	6	$(1,-1,0,1,-1,0,0)$ $(-1,1,1,0,-1,0,0)$ $(0,1,-1,0,1,-1,0)$ $(-1,-1,1,0,1,-1,0)$ $(0,-1,-1,1,0,1,-1)$ $(0,0,0,-1,1,0,1)$
		8	6	$(1,1,0,0,-1,0,0)$ $(1,-1,0,0,1,-1,0)$ $(-1,1,1,-1,1,-1,0)$ $(0,1,-1,0,0,1,-1)$ $(-1,-1,1,0,0,1,-1)$ $(0,-1,-1,1,0,0,1)$

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Nilpotent orbits in type E_7 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of $\mathfrak{g}_i$
		10	6	$(1,1,0,-1,1,-1,0)$ $(1,-1,0,0,0,1,-1)$ $(-1,0,0,1,0,-1,0)$ $(-1,1,1,-1,0,1,-1)$ $(0,1,-1,0,0,0,1)$ $(-1,-1,1,0,0,0,1)$
		12	5	$(1,0,-1,1,0,-1,0)$ $(1,1,0,-1,0,1,-1)$ $(1,-1,0,0,0,0,1)$ $(-1,0,0,1,-1,1,-1)$ $(-1,1,1,-1,0,0,1)$
		14	5	$(0,0,1,0,0,-1,0)$ $(1,0,-1,1,-1,1,-1)$ $(1,1,0,-1,0,0,1)$ $(-1,0,0,0,1,0,-1)$ $(-1,0,0,1,-1,0,1)$
		16	4	$(0,0,1,0,-1,1,-1)$ $(1,0,-1,0,1,0,-1)$ $(1,0,-1,1,-1,0,1)$ $(-1,0,0,0,1,-1,1)$
		18	4	$(0,0,1,-1,1,0,-1)$ $(0,0,1,0,-1,0,1)$ $(1,0,-1,0,1,-1,1)$ $(-1,0,0,0,0,1,0)$
		20	3	$(0,-1,0,1,0,0,-1)$ $(0,0,1,-1,1,-1,1)$ $(1,0,-1,0,0,1,0)$
		22	3	$(0,1,0,0,0,0,-1)$ $(0,-1,0,1,0,-1,1)$ $(0,0,1,-1,0,1,0)$
		24	2	$(0,1,0,0,0,-1,1)$ $(0,-1,0,1,-1,1,0)$
		26	2	$(0,1,0,0,-1,1,0)$ $(0,-1,0,0,1,0,0)$
		28	1	$(0,1,0,-1,1,0,0)$
		30	1	$(0,0,-1,1,0,0,0)$
		32	1	$(-1,0,1,0,0,0,0)$
		34	1	$(1,0,0,0,0,0,0)$

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