

**PREHOMOGENEOUS SPACES ASSOCIATED WITH NILPOTENT
ORBITS IN TYPE EIX**

STEVEN GLENN JACKSON AND ALFRED G. NOËL

Nilpotent orbits in type EIX					
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$	
1.	 $\begin{matrix} & & & & & 0 \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 0 & & 1 & & 1 \end{matrix}$	1	28	$(1, 0, 0, 0, 0, 0, 0, 0)$ $(0, 0, 0, 0, 0, 0, 2)$	
2.	 $\begin{matrix} & & & & & 0 \\ \circ & - & \circ \\ 1 & & 0 & & 0 & & 0 & & 0 & & 0 & & 2 \end{matrix}$	1	32	$(-1, 0, 1, 0, 0, 0, 0, 0)$	
		2	2	$(1, 0, 0, 0, 0, 0, 0, 0)$ $(0, 0, 0, 0, 0, 0, 2)$	
3.	 $\begin{matrix} & & & & & 0 \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 1 & & 0 & & 0 \end{matrix}$	1	32	$(0, 1, 0, 0, 0, -1, 1, 0)$	
		2	10	$(1, 0, 0, 0, 0, 0, 0, 0)$	
4.	 $\begin{matrix} & & & & & 0 \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 0 & & 1 & & 3 \end{matrix}$	1	27	$(1, 0, 0, 0, 0, 0, 0, 0)$	
		3	1	$(0, 0, 0, 0, 0, 0, 0, 2)$	
5.	 $\begin{matrix} & & & & & 0 \\ \circ & - & \circ \\ 1 & & 0 & & 0 & & 0 & & 0 & & 1 & & 1 \end{matrix}$	1	27	$(-1, 0, 0, 0, 0, 1, 0, 0)$ $(0, 1, 0, 0, 0, 0, -1, 0)$ $(0, 0, 0, 0, 0, 0, 2)$	
		2	16	$(-1, 0, 1, 0, 0, 0, 0, 0)$	
		3	1	$(1, 0, 0, 0, 0, 0, 0, 0)$	
6.	 $\begin{matrix} & & & & & 0 \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 4 \end{matrix}$	4	1	$(0, 0, 0, 0, 0, 0, 0, 2)$	
7.	 $\begin{matrix} & & & & & 0 \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 0 & & 2 & & 2 \end{matrix}$	2	28	$(1, 0, 0, 0, 0, 0, 0, 0)$ $(0, 0, 0, 0, 0, 0, 2)$	
8.	 $\begin{matrix} & & & & & 0 \\ \circ & - & \circ \\ 2 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 \end{matrix}$	2	32	$(-1, 0, 1, 0, 0, 0, 0, 0)$	
		4	1	$(1, 0, 0, 0, 0, 0, 0, 0)$	
9.	 $\begin{matrix} & & & & & 1 \\ \circ & - & \circ \\ 1 & & 0 & & 0 & & 0 & & 0 & & 0 & & 1 \end{matrix}$	1	22	$(1, -1, 0, 0, 0, 0, 1, 0)$ $(-1, 0, 0, 0, 0, 1, 0, 0)$ $(0, 0, 0, 0, 0, 0, 2)$	
		2	20	$(0, -1, 0, 0, 1, 0, 0, 0)$	
		3	6	$(-1, 0, 1, 0, 0, 0, 0, 0)$	
		4	1	$(1, 0, 0, 0, 0, 0, 0, 0)$	

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Key words and phrases. Lie group, nilpotent orbit, prehomogeneous space.

Nilpotent orbits in type EIX (continued)				
Orbit	K_C diagram	i	$\dim \mathfrak{g}_C^i \cap \mathfrak{k}_C$	Highest weights of $\mathfrak{g}_C^i \cap \mathfrak{k}_C$
10.		1	24	(0, 0, 1, 0, 0, -1, 0, 0) (-1, 0, 0, 0, 1, -1, 1, 0)
		2	18	(-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0) (0, 0, 0, 0, 0, 0, 2)
		3	8	(-1, 0, 1, 0, 0, 0, 0, 0)
		4	1	(1, 0, 0, 0, 0, 0, 0, 0)
11.		1	24	(1, 1, 0, -1, 0, 0, 1, 0)
		2	18	(0, 0, 1, -1, 0, 1, 0, 0)
		3	8	(0, 1, 0, -1, 1, 0, 0, 0)
		4	3	(1, 0, 0, 0, 0, 0, 0, 0)
12.		1	16	(0, 1, 0, 0, 0, 0, -1, 0)
		2	10	(-1, 0, 0, 0, 0, 1, 0, 0)
		3	16	(-1, 0, 1, 0, 0, 0, 0, 0)
		4	2	(1, 0, 0, 0, 0, 0, 0, 0) (0, 0, 0, 0, 0, 0, 2)
13.		1	16	(0, 1, 0, 0, 0, 0, -1, 0)
		2	2	(0, 0, 0, 0, 0, -1, 2, 0) (0, 0, 0, 0, 0, 0, 2)
		3	16	(0, 1, 0, 0, 0, -1, 1, 0)
		4	10	(1, 0, 0, 0, 0, 0, 0, 0)
14.		2	32	(0, 1, 0, 0, 0, -1, 1, 0)
		4	10	(1, 0, 0, 0, 0, 0, 0, 0)
15.		1	18	(0, 0, 0, 0, -1, 2, 0) (0, 0, 1, 0, 0, -1, 0, 0) (-1, 0, 0, 0, 1, 0, -1, 0) (0, 0, 0, 0, 0, 0, 2)
		2	16	(-1, 0, 0, 0, 1, -1, 1, 0) (0, 1, 0, 0, 0, -1, 0)
		3	9	(-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0)
		4	8	(-1, 0, 1, 0, 0, 0, 0, 0)
		5	1	(1, 0, 0, 0, 0, 0, 0, 0)
16.		1	17	(0, 0, 0, 0, -1, 2, 0) (0, 0, 1, 0, 0, -1, 0, 0) (-1, 0, 0, 0, 1, 0, -1, 0)
		2	16	(-1, 0, 0, 0, 1, -1, 1, 0) (0, 1, 0, 0, 0, 0, -1, 0)
		3	10	(-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0) (0, 0, 0, 0, 0, 0, 2)

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Nilpotent orbits in type EIX (continued)				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$
		4	8	($-1, 0, 1, 0, 0, 0, 0, 0$) ($1, 0, 0, 0, 0, 0, 0, 0$)
17.		1	17	($0, 1, 0, 0, 0, 0, -1, 0$) ($0, 0, 0, 0, 0, 0, 0, 2$)
		3	10	($-1, 0, 0, 0, 0, 1, 0, 0$)
		4	16	($-1, 0, 1, 0, 0, 0, 0, 0$)
		5	1	($1, 0, 0, 0, 0, 0, 0, 0$)
18.		2	32	($0, 1, 0, 0, 0, -1, 1, 0$)
		4	11	($1, 0, 0, 0, 0, 0, 0, 0$) ($0, 0, 0, 0, 0, 0, 0, 2$)
19.		4	27	($1, 0, 0, 0, 0, 0, 0, 0$)
20.		2	27	($-1, 0, 0, 0, 0, 1, 0, 0$) ($0, 1, 0, 0, 0, 0, -1, 0$) ($0, 0, 0, 0, 0, 0, 0, 2$)
		4	16	($-1, 0, 1, 0, 0, 0, 0, 0$)
		6	1	($1, 0, 0, 0, 0, 0, 0, 0$)
21.		4	27	($1, 0, 0, 0, 0, 0, 0, 0$)
		8	1	($0, 0, 0, 0, 0, 0, 0, 2$)
22.		2	16	($0, 1, 0, 0, 0, 0, -1, 0$)
		4	11	($-1, 0, 0, 0, 0, 1, 0, 0$) ($0, 0, 0, 0, 0, 0, 0, 2$)
		6	16	($-1, 0, 1, 0, 0, 0, 0, 0$)
		8	1	($1, 0, 0, 0, 0, 0, 0, 0$)
23.		1	16	($0, 1, -1, 0, 0, 1, -1, 0$) ($0, -1, -1, 1, 0, 0, 1, 0$) ($1, -1, 0, 0, 1, -1, 0$)
		2	16	($0, 1, -1, 0, 0, 0, 1, 0$) ($1, -1, 0, 0, 0, 0, 1, 0$) ($1, 0, -1, 0, 1, 0, -1, 0$) ($0, 0, 0, 0, 0, 0, 0, 2$)
		3	12	($0, -1, 0, 1, 0, 0, -1, 0$) ($1, 0, -1, 0, 0, 1, 0, 0$)
		4	7	($0, 1, 0, 0, 0, 0, -1, 0$) ($0, -1, 0, 0, 1, 0, 0, 0$)
		5	4	($0, 0, -1, 1, 0, 0, 0, 0$)
		6	2	($1, 0, 0, 0, 0, 0, 0, 0$)
24.		4	33	($-1, 0, 1, 0, 0, 0, 0, 0$) ($0, 0, 0, 0, 0, 0, 0, 2$)
		8	1	($1, 0, 0, 0, 0, 0, 0, 0$)

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Nilpotent orbits in type EIX (continued)				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$
25.		2	24	(0, 0, 1, 0, 0, -1, 0, 0) (-1, 0, 0, 0, 1, -1, 1, 0)
		4	17	(-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0)
		6	8	(-1, 0, 1, 0, 0, 0, 0, 0)
		8	1	(1, 0, 0, 0, 0, 0, 0, 0)
26.		1	13	(2, 0, -1, 0, 0, 0, 0, 0) (0, 0, 0, 0, -1, 2, 0) (0, 1, -1, 0, 0, 1, -1, 0) (-1, 0, 0, 1, 0, -1, 0, 0) (0, 0, 0, 0, 0, 0, 2)
		2	14	(0, 1, -1, 0, 0, 0, 1, 0) (1, 0, -1, 1, 0, -1, 0, 0) (-1, 0, 0, 0, 1, 0, -1, 0)
		3	9	(0, 0, 1, 0, 0, -1, 0, 0) (1, 0, -1, 0, 1, 0, -1, 0) (-1, 0, 0, 0, 1, -1, 1, 0)
		4	9	(1, 0, -1, 0, 1, -1, 1, 0) (-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, 0, -1, 0)
		5	5	(1, 0, -1, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0)
		6	6	(0, 0, -1, 1, 0, 0, 0, 0)
		7	1	(-1, 0, 1, 0, 0, 0, 0, 0)
		8	1	(1, 0, 0, 0, 0, 0, 0, 0)
27.		1	12	(0, 1, -1, 0, 0, 1, -1, 0) (1, -1, 0, 0, 0, 1, -1, 0)
		2	12	(1, 0, -1, 0, 1, 0, -1, 0)
		3	8	(0, -1, -1, 1, 0, 0, 1, 0) (0, -1, 0, 1, 0, 0, -1, 0)
		4	5	(0, 1, -1, 0, 0, 0, 1, 0) (1, -1, 0, 0, 0, 0, 1, 0) (0, 1, 0, 0, 0, 0, -1, 0) (0, 0, 0, 0, 0, 0, 2)
		5	8	(1, 0, -1, 0, 0, 1, 0, 0)
		6	6	(0, -1, 0, 0, 1, 0, 0, 0)
		7	4	(0, 0, -1, 1, 0, 0, 0, 0)
		8	2	(1, 0, 0, 0, 0, 0, 0, 0)
28.		2	24	(1, 1, 0, -1, 0, 0, 1, 0)
		4	18	(0, 0, 1, -1, 0, 1, 0, 0)
		6	8	(0, 1, 0, -1, 1, 0, 0, 0)
		8	3	(1, 0, 0, 0, 0, 0, 0, 0)
29.		1	9	(0, 0, 0, 0, 0, -1, 2, 0) (0, 0, 1, 0, 0, -1, 0, 0)

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Nilpotent orbits in type EIX (continued)				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$
30.		3	9	(-1, 0, 0, 0, 1, 0, -1, 0) (0, 0, 0, 0, 0, 0, 2)
		4	16	(-1, 0, 0, 0, 1, -1, 1, 0) (0, 1, 0, 0, 0, 0, -1, 0)
		5	8	(0, 1, 0, 0, 0, -1, 1, 0)
		7	1	(-1, 0, 0, 0, 0, 1, 0, 0)
		8	8	(-1, 0, 1, 0, 0, 0, 0, 0)
		9	1	(1, 0, 0, 0, 0, 0, 0, 0)
31.		2	18	(0, 0, 0, 0, -1, 2, 0) (0, 0, 1, 0, 0, -1, 0, 0) (-1, 0, 0, 0, 1, 0, -1, 0) (0, 0, 0, 0, 0, 0, 2)
		4	16	(-1, 0, 0, 0, 1, -1, 1, 0) (0, 1, 0, 0, 0, 0, -1, 0)
		6	9	(-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0)
		8	8	(-1, 0, 1, 0, 0, 0, 0, 0)
		10	1	(1, 0, 0, 0, 0, 0, 0, 0)
32.		4	33	(0, 1, 0, 0, 0, -1, 1, 0) (0, 0, 0, 0, 0, 0, 0, 2)
		8	10	(1, 0, 0, 0, 0, 0, 0, 0)
		2	16	(0, 0, 1, 0, 0, -1, 0, 0) (-1, 0, 0, 0, 1, 0, -1, 0)
		4	10	(0, 0, 0, 0, 0, -1, 2, 0) (0, 1, 0, 0, 0, 0, -1, 0) (0, 0, 0, 0, 0, 0, 2)
		6	8	(-1, 0, 0, 0, 1, -1, 1, 0)
		8	9	(-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0)
33.		10	8	(-1, 0, 1, 0, 0, 0, 0, 0)
		12	1	(1, 0, 0, 0, 0, 0, 0, 0)
		4	26	(-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, 0, -1, 0)
34.		8	17	(-1, 0, 1, 0, 0, 0, 0, 0) (0, 0, 0, 0, 0, 0, 0, 2)
		12	1	(1, 0, 0, 0, 0, 0, 0, 0)
		2	16	(0, 0, 0, -1, 1, 0, 1, 0) (1, 1, 0, -1, 1, -1, 0, 0)
		4	15	(0, 0, 1, 0, 0, -1, 1, 0) (1, 1, 0, -1, 0, 0, 1, 0)
		6	12	(0, 0, 1, -1, 1, -1, 1, 0)
		8	7	(0, 0, 1, -1, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0)

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Nilpotent orbits in type EIX (continued)				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{k}_{\mathbb{C}}$
35.		4	25	(0, 0, 1, 0, 0, -1, 0, 0) (-1, 0, 0, 0, 1, -1, 1, 0) (0, 0, 0, 0, 0, 0, 2)
		8	17	(-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0)
		12	8	(-1, 0, 1, 0, 0, 0, 0, 0)
		16	1	(1, 0, 0, 0, 0, 0, 0, 0)
36.		4	17	(0, 0, 0, 0, -1, 2, 0) (0, 0, 1, 0, 0, -1, 0, 0) (-1, 0, 0, 0, 1, 0, -1, 0)
		8	17	(-1, 0, 0, 0, 1, -1, 1, 0) (0, 1, 0, 0, 0, 0, -1, 0) (0, 0, 0, 0, 0, 0, 2)
		12	9	(-1, 0, 0, 0, 0, 1, 0, 0) (0, 1, 0, 0, 0, -1, 1, 0)
		16	8	(-1, 0, 1, 0, 0, 0, 0, 0)
		20	1	(1, 0, 0, 0, 0, 0, 0, 0)

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MASSACHUSETTS, 100 MORRISSEY BOULEVARD,
BOSTON, MA 02125-3393

E-mail address: jackson@math.umb.edu

E-mail address: anoel@math.umb.edu