

**PREHOMOGENEOUS SPACES ASSOCIATED WITH NILPOTENT
ORBITS IN TYPE EIX**

STEVEN GLENN JACKSON AND ALFRED G. NOËL

Nilpotent orbits in type EIX				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$
1.	 $\begin{matrix} & & 0 \\ & & \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 0 & & 1 & & 1 \end{matrix}$	1	28	(0, 0, 0, 0, 1, -1, 1) (0, 0, 0, 0, 0, 1, -1)
		2	1	(0, 0, 0, 0, 0, 1, 1)
2.	 $\begin{matrix} & & 0 \\ & & \\ \circ & - & \circ \\ 1 & & 0 & & 0 & & 0 & & 0 & & 0 & & 2 \end{matrix}$	1	32	(-1, 1, 0, 0, 0, 0, 0, 1)
		2	12	(0, 0, 0, 0, 0, 1, 1)
3.	 $\begin{matrix} & & 0 \\ & & \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 1 & & 0 & & 0 \end{matrix}$	1	32	(0, 0, 0, 0, 1, -1, 0, 1)
		2	4	(0, 0, 0, 0, 0, 0, 1, 1)
4.	 $\begin{matrix} & & 0 \\ & & \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 0 & & 1 & & 3 \end{matrix}$	1	27	(1, 0, 0, 0, 0, 0, -1, 1)
		2	27	(0, 0, 0, 0, 0, 1, -1, 1)
		3	1	(0, 0, 0, 0, 0, 0, 1, 1)
5.	 $\begin{matrix} & & 0 \\ & & \\ \circ & - & \circ \\ 1 & & 0 & & 0 & & 0 & & 0 & & 1 & & 1 \end{matrix}$	1	27	(1, 0, 0, 0, 0, 0, -1, 1) (-1, 1, 0, 0, 0, 0, 0, 1) (0, 0, 0, 0, 0, 1, -1, -1)
		2	11	(0, 0, 0, 0, 0, 1, -1, 1) (0, 0, 0, 0, 0, 0, 1, -1)
		3	1	(0, 0, 0, 0, 0, 0, 1, 1)
6.	 $\begin{matrix} & & 0 \\ & & \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 0 & & 0 & & 4 \end{matrix}$	2	56	(0, 0, 0, 0, 0, 0, 1, 1)
7.	 $\begin{matrix} & & 0 \\ & & \\ \circ & - & \circ \\ 0 & & 0 & & 0 & & 0 & & 0 & & 2 & & 2 \end{matrix}$	2	28	(0, 0, 0, 0, 0, 1, -1, 1) (0, 0, 0, 0, 0, 0, 1, -1)
		4	1	(0, 0, 0, 0, 0, 0, 1, 1)
8.	 $\begin{matrix} & & 0 \\ & & \\ \circ & - & \circ \\ 2 & & 0 & & 0 & & 0 & & 0 & & 0 & & 0 \end{matrix}$	2	24	(0, 0, 0, 0, 0, 0, 1, 1)
9.	 $\begin{matrix} & & 1 \\ & & \\ \circ & - & \circ \\ 1 & & 0 & & 0 & & 0 & & 0 & & 0 & & 1 \end{matrix}$	1	22	(-1, -1, 0, 1, 0, 0, 0, 1) (0, -1, 1, 0, 0, 0, 0, -1) (-1, 1, 0, 0, 0, 0, 0, -1)

(continued on next page)

2000 *Mathematics Subject Classification.* 17B05;17B10;17B20;22E30.

Key words and phrases. Lie group, nilpotent orbit, prehomogeneous space.

Nilpotent orbits in type EIX (continued)				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$
		2	13	(−1, 1, 0, 0, 0, 0, 1) (0, −1, 1, 0, 0, 0, 1) (0, 0, 0, 0, 0, 1, −1)
		3	6	(0, 0, 0, 0, 0, 1, 1)
10.		1	24	(−1, 0, 1, 0, 0, −1, 1, 1) (0, 0, 0, 0, 1, −1, 0, −1)
		2	12	(1, 0, 0, 0, 0, −1, 1, 1) (−1, 1, 0, 0, 0, 0, 0, 1) (0, 0, 0, 0, 0, 1, −1)
		3	8	(0, 0, 0, 0, 1, −1, 0, 1)
		4	2	(0, 0, 0, 0, 0, 0, 1, 1)
11.		1	24	(1, 0, 0, −1, 1, 0, 0, 1)
		2	12	(0, 1, 1, −1, 0, 0, 0, 1)
		3	8	(0, 0, 0, 0, 0, 0, 1, 1)
12.		1	16	(−1, 0, 1, 0, 0, 0, −1, 1)
		2	3	(−1, 0, 0, 0, 0, 0, 1, 1) (1, 0, 0, 0, 0, 0, −1, 1) (0, 0, 0, 0, 0, 1, −1)
		3	16	(−1, 1, 0, 0, 0, 0, 0, 1)
		4	10	(0, 0, 0, 0, 0, 1, −1, 1)
		6	1	(0, 0, 0, 0, 0, 0, 1, 1)
13.		1	16	(0, 0, 0, 0, 1, −1, 0, −1)
		2	11	(1, 0, 0, 0, 0, −1, 1, 1) (0, 0, 0, 0, 1, −1, −1)
		3	16	(0, 0, 0, 0, 1, −1, 0, 1)
		4	2	(0, 0, 0, 0, 0, 1, −1, 1) (0, 0, 0, 0, 0, 0, 1, −1)
		6	1	(0, 0, 0, 0, 0, 0, 1, 1)
14.		2	32	(0, 0, 0, 0, 1, −1, 0, 1)
		4	4	(0, 0, 0, 0, 0, 0, 1, 1)
15.		1	18	(1, 0, 0, 0, 0, 0, −1, 1) (−1, 0, 1, 0, 0, −1, 1, 1) (−1, 1, 0, 0, 0, 0, 0, −1) (1, 0, 0, 0, 0, −1, 1, −1)
		2	17	(1, 0, 0, 0, 0, −1, 1, 1) (−1, 1, 0, 0, 0, 0, 0, 1) (0, 0, 0, 0, 1, −1, 0, −1)
		3	9	(0, 0, 0, 0, 1, −1, 0, 1) (0, 0, 0, 0, 0, 1, −1, −1)
		4	2	(0, 0, 0, 0, 0, 1, −1, 1) (0, 0, 0, 0, 0, 0, 1, −1)
(continued on next page)				

Nilpotent orbits in type EIX (continued)				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$
16.		1	17	(-1, 0, 0, 0, 0, 1, 1) (-1, 0, 1, 0, 0, -1, 1) (0, 0, 0, 1, -1, 0, -1)
		2	10	(1, 0, 0, 0, 0, -1, 1) (-1, 0, 1, 0, 0, -1, 1, 1) (0, 0, 0, 0, 1, -1, -1)
		3	10	(1, 0, 0, 0, 0, -1, 1, 1) (-1, 1, 0, 0, 0, 0, 1) (0, 0, 0, 0, 0, 1, -1)
		4	8	(0, 0, 0, 1, -1, 0, 1)
		5	1	(0, 0, 0, 0, 1, -1, 1)
		6	1	(0, 0, 0, 0, 0, 1, 1)
17.		1	17	(-1, 0, 0, 0, 0, 1, 1) (-1, 1, 0, 0, 0, 0, -1)
		2	26	(-1, 1, 0, 0, 0, 0, 1) (0, 0, 0, 0, 1, -1, -1)
		3	10	(0, 0, 0, 0, 1, -1, 1)
		5	1	(0, 0, 0, 0, 0, 1, -1)
		6	1	(0, 0, 0, 0, 0, 0, 1, 1)
18.		2	22	(1, 0, 0, 0, 0, -1, 1, 1) (0, 0, 0, 0, 0, 1, -1)
		4	16	(0, 0, 0, 1, -1, 0, 1)
		6	2	(0, 0, 0, 0, 0, 1, 1)
19.		2	54	(0, 0, 0, 0, 0, 1, -1, 1)
		6	2	(0, 0, 0, 0, 0, 0, 1, 1)
20.		2	27	(1, 0, 0, 0, 0, -1, 1) (-1, 1, 0, 0, 0, 0, 0, 1) (0, 0, 0, 0, 1, -1, -1)
		4	11	(0, 0, 0, 0, 1, -1, 1) (0, 0, 0, 0, 0, 1, -1)
		6	1	(0, 0, 0, 0, 0, 0, 1, 1)
21.		2	28	(1, 0, 0, 0, 0, 0, -1, 1) (0, 0, 0, 0, 0, 1, -1)
		6	27	(0, 0, 0, 0, 0, 1, -1, 1)
		10	1	(0, 0, 0, 0, 0, 0, 1, 1)
22.		2	12	(-1, 0, 0, 0, 0, 0, 1, 1) (1, 0, 0, 0, 0, -1, 1) (0, 0, 0, 0, 1, -1, -1)
		4	16	(-1, 1, 0, 0, 0, 0, 0, 1)
		6	11	(0, 0, 0, 0, 0, 1, -1, 1) (0, 0, 0, 0, 0, 0, 1, -1)
		10	1	(0, 0, 0, 0, 0, 0, 1, 1)

(continued on next page)

Nilpotent orbits in type EIX (continued)				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$
23.		1	16	(0, 0, -1, 1, 0, 0, -1, 1) (0, -1, 0, 0, 0, 1, 0, 1) (1, -1, -1, 1, 0, 0, 0, -1)
		2	11	(1, 0, 0, 0, 0, 0, -1, 1) (0, 0, -1, 0, 1, 0, 0, 1) (0, -1, 1, 0, 0, 0, 0, -1) (1, 1, -1, 0, 0, 0, 0, -1)
		3	12	(1, -1, -1, 1, 0, 0, 0, 1) (0, 0, 0, 0, 1, -1, -1)
		4	4	(1, 1, -1, 0, 0, 0, 0, 1) (0, -1, 1, 0, 0, 0, 0, 1) (0, 0, 0, 0, 0, 1, -1)
		5	4	(0, 0, 0, 0, 1, -1, 1)
		6	1	(0, 0, 0, 0, 0, 1, 1)
24.		2	44	(-1, 1, 0, 0, 0, 0, 0, 1) (0, 0, 0, 0, 0, 0, 1, -1)
		6	12	(0, 0, 0, 0, 0, 1, 1)
25.		2	20	(1, 0, 0, 0, 0, -1, 1, 1) (-1, 1, 0, 0, 0, 0, 0, 1)
		4	16	(0, 0, 0, 0, 1, -1, 0, 1)
		6	4	(0, 0, 0, 0, 0, 0, 1, 1)
26.		1	13	(-1, 0, 1, 0, 0, 0, -1, 1) (0, 0, -1, 1, 0, -1, 1, 1) (0, 0, -1, 0, 1, 0, 0, -1) (-1, 0, 1, 0, 0, -1, 1, -1) (1, 0, 0, 0, 0, -1, -1)
		2	11	(1, 0, 0, 0, 0, -1, 1) (-1, 0, 1, 0, 0, -1, 1, 1) (0, 0, -1, 0, 1, 0, 0, 1) (-1, 1, 0, 0, 0, 0, 0, -1) (1, 0, 0, 0, 0, -1, 1, -1)
		3	9	(1, 0, 0, 0, -1, 1, 1) (-1, 1, 0, 0, 0, 0, 0, 1) (1, 1, -1, 0, 0, 0, 0, -1)
		4	8	(1, 1, -1, 0, 0, 0, 0, 1) (0, 0, 0, 0, 1, -1, 0, -1)
		5	5	(0, 0, 0, 0, 1, -1, 0, 1) (0, 0, 0, 0, 0, 1, -1, -1)
		6	2	(0, 0, 0, 0, 0, 1, -1, 1) (0, 0, 0, 0, 0, 0, 1, -1)
		7	1	(0, 0, 0, 0, 0, 0, 1, 1)
27.		1	12	(0, 0, -1, 1, 0, 0, -1, 1) (1, -1, -1, 1, 0, 0, 0, -1)
		2	7	(1, 0, -1, 0, 0, 0, 1, 1) (1, 0, 0, 0, 0, 0, -1, 1) (0, -1, 1, 0, 0, 0, 0, -1) (1, 1, -1, 0, 0, 0, 0, -1)

(continued on next page)

Nilpotent orbits in type EIX (continued)				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$
		3	8	(0, -1, 0, 0, 1, 0, 1) (0, 0, 0, 0, 1, -1, -1)
		4	6	(0, 0, -1, 0, 1, 0, 0, 1)
		5	8	(1, -1, -1, 1, 0, 0, 0, 1)
		6	4	(1, 1, -1, 0, 0, 0, 0, 1) (0, -1, 1, 0, 0, 0, 0, 1) (0, 0, 0, 0, 0, 1, -1)
		7	4	(0, 0, 0, 0, 1, -1, 1)
		10	1	(0, 0, 0, 0, 0, 1, 1)
28.		2	24	(1, 0, 0, -1, 1, 0, 0, 1)
		4	12	(0, 1, 1, -1, 0, 0, 0, 1)
		6	8	(0, 0, 0, 0, 0, 1, 1)
29.		1	9	(-1, 0, 0, 0, 0, 1, 1) (-1, 0, 1, 0, 0, 0, -1, 1)
		2	17	(1, 0, 0, 0, 0, -1, 1) (-1, 0, 1, 0, 0, -1, 1, 1) (-1, 1, 0, 0, 0, 0, -1)
		3	9	(1, 0, 0, 0, -1, 1, 1) (0, 0, 0, 1, -1, 0, -1)
		5	8	(-1, 1, 0, 0, 0, 0, 1)
		6	9	(0, 0, 0, 1, -1, 0, 1) (0, 0, 0, 0, 1, -1, -1)
		7	1	(0, 0, 0, 0, 0, 1, -1)
		9	1	(0, 0, 0, 0, 1, -1, 1)
		10	1	(0, 0, 0, 0, 0, 1, 1)
30.		2	18	(1, 0, 0, 0, 0, -1, 1) (-1, 0, 1, 0, 0, -1, 1, 1) (-1, 1, 0, 0, 0, 0, 0, -1) (1, 0, 0, 0, -1, 1, -1)
		4	17	(1, 0, 0, 0, -1, 1, 1) (-1, 1, 0, 0, 0, 0, 0, 1) (0, 0, 0, 1, -1, 0, -1)
		6	9	(0, 0, 0, 1, -1, 0, 1) (0, 0, 0, 0, 1, -1, -1)
		8	2	(0, 0, 0, 0, 1, -1, 1) (0, 0, 0, 0, 0, 1, -1)
		10	1	(0, 0, 0, 0, 0, 1, 1)
31.		2	36	(1, 0, 0, 0, 0, -1, 1, 1) (0, 0, 0, 0, 1, -1, 0, -1)
		6	18	(0, 0, 0, 0, 1, -1, 0, 1) (0, 0, 0, 0, 0, 0, 1, -1)
		10	2	(0, 0, 0, 0, 0, 0, 1, 1)
32.		2	11	(-1, 0, 0, 0, 0, 0, 1, 1) (1, 0, 0, 0, 0, -1, 1) (-1, 1, 0, 0, 0, 0, 0, -1) (1, 0, 0, 0, 0, -1, 1, -1)
(continued on next page)				

Nilpotent orbits in type EIX (continued)				
Orbit	$K_{\mathbb{C}}$ diagram	i	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$
33.		4	16	(-1, 0, 1, 0, 0, -1, 1, 1) (0, 0, 0, 1, -1, 0, -1)
		6	10	(1, 0, 0, 0, 0, -1, 1, 1) (-1, 1, 0, 0, 0, 0, 0, 1) (0, 0, 0, 0, 1, -1, -1)
		8	8	(0, 0, 0, 0, 1, -1, 0, 1)
		10	2	(0, 0, 0, 0, 1, -1, 1) (0, 0, 0, 0, 0, 1, -1)
		14	1	(0, 0, 0, 0, 0, 1, 1)
34.		2	27	(-1, 0, 0, 0, 0, 0, 1, 1) (-1, 0, 1, 0, 0, 0, -1, 1) (0, 0, 0, 0, 1, -1, -1)
		6	18	(1, 0, 0, 0, 0, 0, -1, 1) (-1, 1, 0, 0, 0, 0, 0, 1) (0, 0, 0, 0, 0, 1, -1)
		10	10	(0, 0, 0, 0, 1, -1, 1)
		14	1	(0, 0, 0, 0, 0, 1, 1)
		2	16	(0, 1, 0, -1, 0, 1, 0, 1) (1, 0, 0, 0, 0, -1, 1, 1)
35.		4	12	(1, 0, 0, -1, 1, 0, 0, 1)
		6	12	(0, 1, 1, -1, 0, 0, 0, 1)
		8	4	(0, 0, 0, 1, -1, 0, 1)
		10	4	(0, 0, 0, 0, 0, 1, 1)
		2	26	(-1, 0, 1, 0, 0, -1, 1, 1) (-1, 1, 0, 0, 0, 0, 0, -1) (1, 0, 0, 0, 0, -1, 1, -1)
36.		6	18	(1, 0, 0, 0, 0, -1, 1, 1) (-1, 0, 1, 0, 0, 0, -1, 1) (-1, 1, 0, 0, 0, 0, 0, -1) (1, 0, 0, 0, 0, -1, 1, -1)
		10	10	(0, 0, 0, 0, 1, -1, 0, 1) (0, 0, 0, 0, 0, 0, 1, -1)
		14	9	(0, 0, 0, 0, 1, -1, 0, 1) (0, 0, 0, 0, 0, 0, 1, -1)
		18	1	(0, 0, 0, 0, 0, 1, -1, 1)
		22	1	(0, 0, 0, 0, 0, 0, 1, 1)

PREHOMOGENEOUS SPACES ASSOCIATED WITH NILPOTENT ORBITS IN TYPE **EIX** 7

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MASSACHUSETTS, 100 MORRISSEY BOULEVARD,
BOSTON, MA 02125-3393

E-mail address: jackson@math.umb.edu

E-mail address: anoel@math.umb.edu