

**PREHOMOGENEOUS SPACES ASSOCIATED WITH NILPOTENT
ORBITS IN $F_4(\mathbb{C})$**

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Nilpotent orbits in type F_4				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of \mathfrak{g}_i
A_1	$\begin{array}{ccccc} \circ & -\!\!-\!\!-\! & \circ & -\!\!-\!\!-\! & \circ \\ 1 & 0 & 0 & 0 \end{array}$	1	14	$(-1, 1, 0, 0)$
		2	1	$(1, 0, 0, 0)$
\tilde{A}_1	$\begin{array}{ccccc} \circ & -\!\!-\!\!-\! & \circ & -\!\!-\!\!-\! & \circ \\ 0 & 0 & 0 & 1 \end{array}$	1	8	$(0, 0, 1, -1)$
		2	7	$(1, 0, 0, 0)$
$A_1 + \tilde{A}_1$	$\begin{array}{ccccc} \circ & -\!\!-\!\!-\! & \circ & -\!\!-\!\!-\! & \circ \\ 0 & 1 & 0 & 0 \end{array}$	1	12	$(1, -1, 0, 2)$
		2	6	$(0, -1, 2, 0)$
		3	2	$(1, 0, 0, 0)$
A_2	$\begin{array}{ccccc} \circ & -\!\!-\!\!-\! & \circ & -\!\!-\!\!-\! & \circ \\ 2 & 0 & 0 & 0 \end{array}$	2	14	$(-1, 1, 0, 0)$
		4	1	$(1, 0, 0, 0)$
\tilde{A}_2	$\begin{array}{ccccc} \circ & -\!\!-\!\!-\! & \circ & -\!\!-\!\!-\! & \circ \\ 0 & 0 & 0 & 2 \end{array}$	2	8	$(0, 0, 1, -1)$
		4	7	$(1, 0, 0, 0)$
$A_2 + \tilde{A}_1$	$\begin{array}{ccccc} \circ & -\!\!-\!\!-\! & \circ & -\!\!-\!\!-\! & \circ \\ 0 & 0 & 1 & 0 \end{array}$	1	6	$(1, 0, -1, 1)$
		2	9	$(0, 1, -2, 2)$
		3	2	$(0, 0, 0, 1)$
		4	3	$(1, 0, 0, 0)$
B_2	$\begin{array}{ccccc} \circ & -\!\!-\!\!-\! & \circ & -\!\!-\!\!-\! & \circ \\ 2 & 0 & 0 & 1 \end{array}$	1	4	$(-1, 0, 1, 0)$
		2	6	$(0, 1, 0, -2)$ $(-1, 0, 0, 2)$
		3	4	$(0, 0, 1, -1)$
		4	5	$(-1, 1, 0, 0)$
		6	1	$(1, 0, 0, 0)$
$\tilde{A}_2 + A_1$	$\begin{array}{ccccc} \circ & -\!\!-\!\!-\! & \circ & -\!\!-\!\!-\! & \circ \\ 0 & 1 & 0 & 1 \end{array}$	1	8	$(0, -1, 1, 1)$ $(1, -1, 2, -2)$
		2	5	$(0, 1, 0, -2)$ $(1, -1, 1, 0)$
		3	4	$(1, -1, 0, 2)$ $(0, 0, 1, -1)$
		4	3	$(0, -1, 2, 0)$
<i>(continued on next page)</i>				

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Nilpotent orbits in type F_4 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of \mathfrak{g}_i
		5	2	(1, 0, 0, 0)
$C_3(a_1)$	$\begin{array}{ccccc} \circ & -\circ & \nearrow & \circ & -\circ \\ & 1 & 0 & 1 & 0 \end{array}$	1	6	(1, 1, -2, 0) (-1, 1, -1, 1)
		2	5	(1, 0, -1, 1) (-1, 0, 0, 2)
		3	6	(0, 1, -2, 2)
		4	2	(0, 0, 0, 1)
		5	2	(-1, 1, 0, 0)
		6	1	(1, 0, 0, 0)
$F_4(a_3)$	$\begin{array}{ccccc} \circ & -\circ & \nearrow & \circ & -\circ \\ & 0 & 2 & 0 & 0 \end{array}$	2	12	(1, -1, 0, 2)
		4	6	(0, -1, 2, 0)
		6	2	(1, 0, 0, 0)
B_3	$\begin{array}{ccccc} \circ & -\circ & \nearrow & \circ & -\circ \\ & 2 & 2 & 0 & 0 \end{array}$	2	7	(2, -1, 0, 0) (-1, 0, 0, 2)
		4	6	(1, -1, 0, 2)
		6	6	(0, -1, 2, 0)
		8	1	(-1, 1, 0, 0)
		10	1	(1, 0, 0, 0)
C_3	$\begin{array}{ccccc} \circ & -\circ & \nearrow & \circ & -\circ \\ & 1 & 0 & 1 & 2 \end{array}$	1	4	(1, 1, -2, 0) (-1, 1, 0, -1)
		2	3	(0, 0, -1, 2) (1, 0, 0, -1) (-1, 0, 2, -2)
		3	4	(-1, 1, -1, 1) (0, 1, 0, -2)
		4	2	(1, 0, -1, 1) (-1, 0, 1, 0)
		5	2	(0, 1, -1, 0)
		6	2	(-1, 0, 0, 2) (0, 0, 1, -1)
		7	2	(0, 1, -2, 2)
		8	1	(0, 0, 0, 1)
		9	2	(-1, 1, 0, 0)
		10	1	(1, 0, 0, 0)
$F_4(a_2)$	$\begin{array}{ccccc} \circ & -\circ & \nearrow & \circ & -\circ \\ & 0 & 2 & 0 & 2 \end{array}$	2	8	(0, -1, 1, 1) (1, -1, 2, -2)
		4	5	(0, 1, 0, -2) (1, -1, 1, 0)
		6	4	(1, -1, 0, 2) (0, 0, 1, -1)
		8	3	(0, -1, 2, 0)
		10	2	(1, 0, 0, 0)
(continued on next page)				

Nilpotent orbits in type F_4 (continued)				
Bala-Carter label	Diagram	i	$\dim \mathfrak{g}_i$	Highest weights of \mathfrak{g}_i
$F_4(a_1)$	 $\begin{matrix} & \circ & - & \circ & \rightleftharpoons & \circ & - & \circ \\ & 2 & & 2 & & 0 & & 2 \end{matrix}$	2	6	(2, -1, 0, 0) (0, -1, 1, 1) (-1, 0, 2, -2)
		4	5	(1, -1, 2, -2) (-1, 0, 1, 0)
		6	4	(0, 1, 0, -2) (1, -1, 1, 0) (-1, 0, 0, 2)
		8	3	(1, -1, 0, 2) (0, 0, 1, -1)
		10	3	(0, -1, 2, 0)
		12	1	(-1, 1, 0, 0)
		14	1	(1, 0, 0, 0)
F_4	 $\begin{matrix} & \circ & - & \circ & \rightleftharpoons & \circ & - & \circ \\ & 2 & & 2 & & 2 & & 2 \end{matrix}$	2	4	(2, -1, 0, 0) (-1, 2, -2, 0) (0, -1, 2, -1) (0, 0, -1, 2)
		4	3	(1, 1, -2, 0) (-1, 1, 0, -1) (0, -1, 1, 1)
		6	3	(1, 0, 0, -1) (-1, 0, 2, -2) (-1, 1, -1, 1)
		8	3	(1, -1, 2, -2) (1, 0, -1, 1) (-1, 0, 1, 0)
		10	3	(0, 1, 0, -2) (1, -1, 1, 0) (-1, 0, 0, 2)
		12	2	(0, 1, -1, 0) (1, -1, 0, 2)
		14	2	(0, 0, 1, -1) (0, 1, -2, 2)
		16	1	(0, 0, 0, 1)
		18	1	(0, -1, 2, 0)
		20	1	(-1, 1, 0, 0)
		22	1	(1, 0, 0, 0)

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