

**PREHOMOGENEOUS SPACES ASSOCIATED WITH NILPOTENT  
ORBITS IN TYPE *FI***

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<b>Nilpotent orbits in type <i>FI</i></b>				
Orbits	Diagram	$i$	$\dim \mathfrak{g}_c^i \cap \mathfrak{p}_c$	Highest weights of $\mathfrak{g}_c^i \cap \mathfrak{p}_c$
1	 $\begin{matrix} 0 & 0 & 1 & 1 \end{matrix}$	1	7	$(0, 2, -1, 1)$ $(0, 0, 1, -1)$
		2	1	$(0, 0, 1, 1)$
2	 $\begin{matrix} 1 & 0 & 0 & 2 \end{matrix}$	1	4	$(-1, 1, 0, 1)$
		2	5	$(0, 0, 1, 1)$
3	 $\begin{matrix} 0 & 1 & 0 & 0 \end{matrix}$	1	4	$(1, 0, 0, 1)$
		2	4	$(0, 0, 1, 1)$
4	 $\begin{matrix} 0 & 0 & 1 & 3 \end{matrix}$	1	6	$(2, 0, -1, 1)$
		2	6	$(0, 2, -1, 1)$
		3	1	$(0, 0, 1, 1)$
5	 $\begin{matrix} 1 & 0 & 1 & 1 \end{matrix}$	1	6	$(2, 0, -1, 1)$ $(-1, 1, 0, 1)$ $(0, 2, -1, -1)$
		2	4	$(0, 2, -1, 1)$ $(0, 0, 1, -1)$
		3	1	$(0, 0, 1, 1)$
6	 $\begin{matrix} 0 & 0 & 0 & 4 \end{matrix}$	2	14	$(0, 0, 1, 1)$
7	 $\begin{matrix} 2 & 0 & 0 & 0 \end{matrix}$	2	10	$(0, 0, 1, 1)$
8	 $\begin{matrix} 0 & 0 & 2 & 2 \end{matrix}$	2	7	$(0, 2, -1, 1)$ $(0, 0, 1, -1)$
		4	1	$(0, 0, 1, 1)$
9	 $\begin{matrix} 0 & 2 & 0 & 0 \end{matrix}$	2	4	$(1, 0, 0, 1)$
		4	4	$(0, 0, 1, 1)$
10	 $\begin{matrix} 1 & 1 & 0 & 2 \end{matrix}$	1	3	$(0, -1, 1, 1)$ $(1, 0, 0, -1)$
		2	5	$(-1, 1, 0, 1)$ $(2, -2, 1, 1)$ $(0, 0, 1, -1)$
		3	1	$(1, 0, 0, 1)$

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*Key words and phrases.* Lie group, nilpotent orbit, prehomogeneous space.

Nilpotent orbits in type $FI$ (continued)				
Orbits	Diagram	$i$	$\dim \mathfrak{g}_c^i \cap \mathfrak{p}_c$	Highest weights of $\mathfrak{g}_c^i \cap \mathfrak{p}_c$
		4	2	(0, 0, 1, 1)
11	 $\begin{matrix} & & & \\ 1 & 0 & 2 & 4 \end{matrix}$	1	2	(0, 1, -1, 1)
		2	3	(-2, 0, 1, 1) (2, 0, -1, 1) (0, 0, 1, -1)
		3	2	(-1, 1, 0, 1)
		4	3	(0, 2, -1, 1)
		6	1	(0, 0, 1, 1)
12	 $\begin{matrix} & & & \\ 0 & 1 & 2 & & 2 \end{matrix}$	1	2	(1, 0, 0, -1)
		2	4	(2, -2, 1, 1) (0, 2, -1, -1)
		3	2	(1, 0, 0, 1)
		4	2	(0, 2, -1, 1) (0, 0, 1, -1)
		6	1	(0, 0, 1, 1)
13	 $\begin{matrix} & & & \\ 1 & 1 & 1 & & 1 \end{matrix}$	1	4	(0, -1, 1, 1) (2, 0, -1, 1) (2, -2, 1, -1) (-1, 1, 0, -1)
		2	3	(-1, 1, 0, 1) (2, -2, 1, 1) (1, 0, 0, -1)
		3	2	(1, 0, 0, 1) (0, 2, -1, -1)
		4	2	(0, 2, -1, 1) (0, 0, 1, -1)
		5	1	(0, 0, 1, 1)
14	 $\begin{matrix} & & & \\ 1 & 0 & 3 & & 1 \end{matrix}$	1	3	(-2, 0, 1, 1) (-1, 1, 0, -1)
		2	5	(-1, 1, 0, 1) (0, 2, -1, -1)
		3	3	(0, 2, -1, 1)
		5	1	(0, 0, 1, -1)
		6	1	(0, 0, 1, 1)
15	 $\begin{matrix} & & & \\ 1 & 1 & 1 & & 3 \end{matrix}$	1	3	(-2, 0, 1, 1) (0, 1, -1, 1) (1, 0, 0, -1)
		2	3	(0, -1, 1, 1) (2, 0, -1, 1) (0, 2, -1, -1)
		3	3	(-1, 1, 0, 1) (2, -2, 1, 1) (0, 0, 1, -1)
		4	1	(1, 0, 0, 1)
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Nilpotent orbits in type <b><i>FI</i></b> (continued)				
Orbits	Diagram	$i$	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$
		5	1	(0, 2, -1, 1)
		6	1	(0, 0, 1, 1)
16	 $\begin{matrix} 0 & 0 & 4 & 0 \end{matrix}$	2	12	(0, 2, -1, 1)
		6	2	(0, 0, 1, 1)
17	 $\begin{matrix} 0 & 2 & 0 & 4 \end{matrix}$	2	8	(2, -2, 1, 1) (0, 0, 1, -1)
		4	2	(1, 0, 0, 1)
		6	2	(0, 0, 1, 1)
18	 $\begin{matrix} 2 & 0 & 2 & 2 \end{matrix}$	2	6	(2, 0, -1, 1) (-1, 1, 0, 1) (0, 2, -1, -1)
		4	4	(0, 2, -1, 1) (0, 0, 1, -1)
		6	1	(0, 0, 1, 1)
19	 $\begin{matrix} 0 & 0 & 4 & 8 \end{matrix}$	2	7	(2, 0, -1, 1) (0, 0, 1, -1)
		6	6	(0, 2, -1, 1)
		10	1	(0, 0, 1, 1)
20	 $\begin{matrix} 2 & 0 & 4 & 4 \end{matrix}$	2	5	(-2, 0, 1, 1) (2, 0, -1, 1) (0, 2, -1, -1)
		4	2	(-1, 1, 0, 1)
		6	4	(0, 2, -1, 1) (0, 0, 1, -1)
		10	1	(0, 0, 1, 1)
21	 $\begin{matrix} 1 & 3 & 1 & 3 \end{matrix}$	1	2	(-2, 0, 1, 1) (0, 1, -1, 1)
		2	3	(0, -1, 1, 1) (2, 0, -1, 1) (-1, 1, 0, -1)
		3	2	(2, -2, 1, 1) (1, 0, 0, -1)
		5	1	(-1, 1, 0, 1)
		6	2	(1, 0, 0, 1) (0, 2, -1, -1)
		7	1	(0, 0, 1, -1)
		9	1	(0, 2, -1, 1)
		10	1	(0, 0, 1, 1)
22	 $\begin{matrix} 0 & 4 & 0 & 4 \end{matrix}$	2	8	(2, -2, 1, 1) (1, 0, 0, -1)
		6	4	(1, 0, 0, 1) (0, 0, 1, -1)
		10	2	(0, 0, 1, 1)
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Nilpotent orbits in type $FI$ (continued)				
Orbits	Diagram	$i$	$\dim \mathfrak{g}_c^i \cap \mathfrak{p}_c$	Highest weights of $\mathfrak{g}_c^i \cap \mathfrak{p}_c$
23		2	4	(0, -1, 1, 1) (2, 0, -1, 1) (2, -2, 1, -1) (-1, 1, 0, -1)
		4	3	(-1, 1, 0, 1) (2, -2, 1, 1) (1, 0, 0, -1)
		6	2	(1, 0, 0, 1) (0, 2, -1, -1)
		8	2	(0, 2, -1, 1) (0, 0, 1, -1)
		10	1	(0, 0, 1, 1)
24		2	4	(-2, 0, 1, 1) (2, 0, -1, 1) (2, -2, 1, -1) (-1, 1, 0, -1)
		4	2	(0, -1, 1, 1) (1, 0, 0, -1)
		6	3	(-1, 1, 0, 1) (2, -2, 1, 1) (0, 2, -1, -1)
		8	1	(1, 0, 0, 1)
		10	2	(0, 2, -1, 1) (0, 0, 1, -1)
		14	1	(0, 0, 1, 1)
25		2	6	(-2, 0, 1, 1) (0, 1, -1, 1) (0, 2, -1, -1)
		6	4	(2, 0, -1, 1) (-1, 1, 0, 1) (0, 0, 1, -1)
		10	3	(0, 2, -1, 1)
		14	1	(0, 0, 1, 1)
26		2	4	(-2, 0, 1, 1) (0, 1, -1, 1) (2, -2, 1, -1) (-1, 1, 0, -1)
		6	3	(0, -1, 1, 1) (2, 0, -1, 1) (1, 0, 0, -1)
		10	3	(-1, 1, 0, 1) (2, -2, 1, 1) (0, 2, -1, -1)
		14	2	(1, 0, 0, 1) (0, 0, 1, -1)
(continued on next page)				

Nilpotent orbits in type <b><i>FI</i></b> (continued)				
Orbits	Diagram	$i$	$\dim \mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$	Highest weights of $\mathfrak{g}_{\mathbb{C}}^i \cap \mathfrak{p}_{\mathbb{C}}$
		18	1	(0, 2, -1, 1)
		22	1	(0, 0, 1, 1)

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