Homework 15
(due December 1)
Math 130 Kovitz 2016

1. Explain the following apparent error in the sin function.

The equation

\[ \sin \left( \frac{\pi}{6} \right) \approx \sin \left( \frac{3.141592654}{6} \right) \approx \sin .523598776 \approx .5 \]

is true.

But (.523598776, .5) is not a point on the unit circle since

\[ (.523598776)^2 + (.5)^2 \approx .274155678 + .25 = .524155678 \neq 1, \]

meaning that

\[ u^2 + v^2 \neq 1 \]

and that the point (.523598776, .5) is not on the unit circle.

The above statement is also true. In fact the only first-quadrant point on the unit circle for which \( v = .5 \) is (.866025404, .5) (approximately).

\textit{How could } \sin(.523598776) \approx .5 \textit{ when the point (.523598776, .5) is not even approximately on the unit circle.} \textit{Explain. (Hint: ask yourself — of what is the unit circle a graph?)}

2. (a) Graph \( \sin x \) and \( \cos x \), for \( x \) between 0 and \( 2\pi \), on the same axes.

Where do they intersect? Label the two points.

(b) Using the unit circle \((u, v) = (\cos s, \sin s)\), find the points where

\[ \sin s = \cos s, \text{ for } 0 \leq s \leq 2\pi. \]

Label them.

(c) Show that the answers in parts (a) and (b) are exactly the same. Explain briefly.

\textbf{TURN OVER}
3. By using a graph of the unit circle draw the points that have the property that \( \cos s = \frac{1}{2} \).

   (a) By looking at the graph, roughly estimate \( s \) for each such point. (Is there more than one answer to this question for each point?)

   (b) By looking at the graph, roughly estimate the value of \( \sin s \) for each such point.

   *(c)* It is surely true that \( \sin s \) is a function—that means that \( \sin s \) cannot have more than one value for a given \( s \).

   Explain the apparent contradiction to that fact by the two points which have the property that \( \cos s = \frac{1}{2} \) but have different sines.

   Also explain the apparent contradiction to \( \cos s \) being a function by the fact that there are two values of \( s \) which make \( \cos s = \frac{1}{2} \) true.

   (d) By using a graph of \( \cos x \), locate some points which have the property that \( \cos x = \frac{1}{2} \). By looking at the graph, roughly estimate \( x \) for at least four of those points.

4. (a) Find the period

   i. \( \cos(7x) \)
   ii. \( \sin(\frac{x}{2}) \)
   iii. \( \cos(2\pi x) \)
   iv. \( \cos(\frac{2\pi x}{3}) \)

   (b) Match to the correct graph

   i. \( \cos(\frac{2\pi x}{3}) \)
   ii. \( \sin(4x) \)

5. By translation, graph \( y = \sin(x - \frac{\pi}{3}) \) for \( 0 \leq x \leq 2\pi \). Label with coordinates all intercepts and the peak and the valley.