Review by Stephen Parrott of
Collective Electrodynamics
by
Carver A. Mead

This is an unusual book and not an easy one to review. Perhaps the best starting place is the publishers' summary, on the back cover, of its intent:

> "In this book Carver Mead offers a radically new approach to the standard problems of electromagnetic theory. Motivated by the belief that the goal of scientific research should be the simplification and unification of knowledge, he describes a new way of doing electrodynamics—collective electrodynamics—that does not rely on Maxwell's equations, but rather uses the quantum nature of matter as its sole basis. Collective electrodynamics is a way of looking at how electrons interact, based on experiments that tell us about the electron directly. (As Mead points out, Maxwell had no access to these experiments.)
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> The results Mead derives for standard electromagnetic problems are identical to those found in any text. Collective electrodynamics reveals, however, that quantities that we usually think of as being very different are, in fact, the same—that electromagnetic phenomena are direct manifestations of quantum phenomena. Mead views this as a first step toward reformulating quantum concepts in a clear and comprehensive manner."

It was this summary that persuaded me to order, sight unseen, this small (132 pages) but relatively inexpensive book to read on vacation. I didn't expect a lot from it, but I hoped that it might furnish some new insights. I was very disappointed that I learned nothing of substance from it.

Indeed, I think that the above summary borders on false advertising. The book does not convincingly obtain classical electrodynamics from accepted quantum mechanical principles nor from experiments to which "Maxwell had no access". Its motivation is presented in such a vague and sloppy way that I regard it as yet one more of the endless accumulation of dreary papers which Pauli, in a famous remark, characterized as "not even wrong", i.e., too vague to be meaningful.

The book only sketchily describes the "experiments that tell us about the electron directly". These are experiments with superconducting coils, which reveal not the behavior of individual electrons, but behavior of a system of a large number of electrons coupled in poorly understood ways (hence the "collective" in the book's title). Most of the book's development is based on just one experimental fact—that the magnetic flux of a superconducting loop is quantized, i.e., the flux can take on only values which are a constant multiple of positive integers. The book views such a system as a primitive system "having only one degree of freedom".

Before proceeding to sketch the book's main argument, I have to make some mathematical remarks. It is well known that classical electrodynamics can be plausibly developed starting with just one mathematical object—the four-potential $A = A_i$, $i = 0, 1, 2, 3$, which is a one-form on four-dimensional Minkowski space. The electromagnetic field tensor F, a 2-form, is the differential of the potential 1-form: $F = dA$. It would take too long to give precise definitions here, but they can be found in my book *Relativistic Electrodynamics and Differential Geometry* and many other places.

The 4-current J is then defined as (or, from a more physical point of view, assumed to be) the codifferential (covariant divergence) of the field tensor. This mathematical structure is equivalent to Maxwell's equations. Thus from any physical situation in which a 1-form on Minkowski space appears naturally, one can plausibly recover a good deal of the mathematical structure of classical electrodynamics. For example, if within the logical structure of thermodynamics there were a naturally occurring 1-form on Minkowski space, one might attempt to "derive" electrodynamics from thermodynamics by identifying this "natural" thermodynamic 1-form with the electromagnetic potential A.

However, there would remain more work to be done. The traditional mathematical structure of classical electrodynamics requires that the four-potential A be the *retarded* solution of d'Alembert's equation $(\partial^2/\partial x_0^2 - \sum_{k=1}^{3} \partial^2/x_k^2)A_i = J_i$ with source J, which below we'll call the "retardation condition". This is an additional constraint on A which might or might not be satisfied by the naturally occurring thermodynamic 1-form which we would like to identify with the four-potential A of electrodynamics. But if our naturally occurring 1-form should be an unmeasurable quantity within thermodynamics, then this problem would not exist. One might just *assume* that it satisfied this additional constraint, and claim to have "derived" electrodynamics from thermodynamics.

When the logic is put together this way, it may sound somewhat silly. It's clear that little, if anything, would be accomplished by such a "derivation" of electrodynamics from thermodynamics. It would be mainly smoke and mirrors. But if the "derivation" were more obscurely written, it might appear possible that something had been accomplished.

The essence of Mead's argument is that the logical structure of quantum mechanics furnishes a naturally occurring 1-form on three-dimensional space with the property that its integral over any closed curve is an integer. The space part $\vec{A}$ of the electromagnetic potential 1-form A has the property that its integral over any superconducting loop gives the magnetic flux threading the loop, which is observed to always be a constant multiple of an integer. This suggests identifying Mead's naturally occurring 1-form with $\vec{A}$. Later, the full electromagnetic potential A is obtained from $\vec{A}$ by hand-waving analogy.

His naturally occurring 1-form is assumed to have the property that integrating it over a closed curve gives the change in phase of an "electron wave function" (which he never properly defines) over the curve. Mead's proposed 1-form from quantum mechanics seems inherently unmeasurable, i.e., there is no way to determine it from experiment (or at least Mead doesn't propose any). Mead does not address the question of whether his "change in phase" 1-form satisfies the retardation condition above; he just seems to assume it. In my opinion, the main problem with his argument is that his construction of his "phase change" 1-form is so vague, sloppy, and problematic that it is "not even wrong".

I shall now attempt to summarize his construction. This is difficult because Mead never properly defines his symbols, so one is never quite sure that one understands what he is talking about. What I shall say is merely my best guess as to the meaning of Mead's vague exposition.

Part 1 of the book starts by describing the quantization of magnetic flux in it's third equation, (1.3). This equation is written in the engineering language of voltage, but readers sufficiently familiar with electrodynamics to hope to read the book will know how to translate it into a statement about quantization of magnetic flux.

Next the book starts talking about the quantum wave function of a superconducting loop. There is no preliminary discussion of quantum mechanics to establish notation and the author's point of view regarding this often controversial subject.

To give the reader a feel for the vagueness of the book's exposition, I quote the relevant parts of this short discussion. A reader who has the book in front of him will recognize that I have omitted nothing which might sharpen the vagueness of the discussion.

> "Electrons in a superconductor are described by a **wave function** that has an amplitude and a phase. The earliest treatment of the wave nature of matter is the 1923 wave mechanics of de Broglie. He applied the Einstein postulate ($W = \hbar\omega$) to the energy W of an electron wave and identified the momentum $\vec{p} = \hbar\vec{k}$. ...
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> "The Einstein-de Broglie relations apply to the collective electrons in a superconductor. ... A more detailed description of the wave function of a large ensemble of electrons is given in the Appendix (p. 115)."

The symbols ω and k, which appear here for the first time in the book, are never defined, here or elsewhere. Is it obvious what ω and $\vec{k}$ might mean in the present context of a superconducting loop? How would one measure ω and $\vec{k}$ for a superconducting loop?

If we look ahead to p. 115 of the Appendix to find out what the author means by the "wave function" of a superconducting loop, the first sentence is:

> "In a solid, the most appropriate electronic states are traveling waves described by a wave function ψ:
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> $$\psi = e^{i(\omega t - k\cdot r)} \quad "$$

Again, the text does not define ω and k, and the author does not tell us why states described by this wave function are "the most appropriate". With an exposition this vague, is it surprising that most anything could be "derived"?

In case the reader imagines that he can guess the meaning of ω and k (e.g., k is a constant vector and $\omega := |k|$) and suspects that I am being overly pedantic in my insistence that all symbols should be clearly defined, please reserve judgment for a few paragraphs. We'll see that the definitions are not at all obvious (and in particular, the guess just made is probably wrong).

Returning to Part 1, p. 12, the book continues:

> "The wave function must be *continuous* in space; at any given time, we can follow the phase along a path from one end of the loop to the other: The number of radians by which the phase advances as we traverse the path is the **phase accumulation** ϕ around the loop. If the phase at one end of the loop changes relative to that of the other end, that change must be reflected in the total phase accumulation around the loop. The **frequency** ω of the wave function at any point in space is the rate at which the phase advances per unit of time. If the frequency at one end of the loop (ω_1) is the same as that at the other end (ω_2), the phase difference between the two ends will remain constant, and the phase accumulation will not change with time. If the frequency at one end of the loop is higher than that at the other, the phase accumulation will increase with time, and that change must be reflected in the rate at which phase accumulates with the distance l along the path. The rate at which the phase around the loop accumulates with time is the difference in frequency between the two ends. The rate at which phase accumulates with distance l is the component of the propagation vector $\vec{k}$ in the direction $\vec{dl}$ along the path. Thus the total phase accumulated along the loop is
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> $$\phi = \int (\omega_1 - \omega_2) dt = \oint \vec{k} \cdot \vec{dl} \quad \text{''}$$

This last equation, the text's (1.4), is the key equation by which the author obtains the naturally occuring "phase-change" 1-form mentioned above, which is later identified with the space part $\vec{A}$ of the electrodynamic potential 1-form A. I'll say more about this in a moment, but first let's critically examine the above equation (1.4) in the light of the text's only explanation of the symbols ω and $\vec{k}$ (all of which has been fully quoted above, so far as I am aware).

I mentioned earlier that the guess that $\vec{k}$ is a constant vector, which seemed natural in the context of the Appendix, is probably the wrong guess. This is because if $\vec{k}$ is constant, then the above integral, $\oint \vec{k} \cdot \vec{dl}$ around a closed loop, obviously *vanishes*. But a few equations later, the text claims that

$$\phi = \oint \vec{k} \cdot \vec{dl} = 2\pi n \quad ,$$

where n is an integer (which is not necessarily zero, as the context makes clear).

So, what does the symbol $\vec{k}$ represent in the above equation (1.4), which is only the fourth and probably the most important equation in the book? Despite considerable thought, I'm still puzzled about this.

My best guess is that $\vec{k}$ may be intended to be a function $\vec{k} = \vec{k}(\vec{x})$ of position $\vec{x}$, in which case $\vec{k} \cdot \vec{dl}$ would represent a 1-form whose integral over a closed curve would not obviously vanish.[1] The text may be *assuming* the existence of a 1-form, which it denotes $\vec{k} \cdot \vec{dl}$, whose line integral over any curve gives the phase change of the system's wave function over that curve.

But there is a serious problem with this "best guess". If the wave function is defined and nonzero on all of space, as is the above $\psi = e^{i(\omega t - kr)}$ which Mead seems to regard as the "most appropriate", then so is its phase (up to an everywhere constant multiple of 2π). Its phase is necessarily a (single-valued) function (e.g., the phase of ψ at time 0 is $-kr$), and the change in a (single-valued) function over a *closed* curve necessarily vanishes. So again, we have $\oint \vec{k} \cdot \vec{dl} = 0$ in contradiction to the text's above-mentioned claim that $\oint \vec{k} \cdot \vec{dl} = 2\pi n$ with n not necessarily zero.

If the wave function can vanish or may be not be everywhere defined, the situation could be different, but the text never discusses these possibilities. In the absence of discussions of such important points, the book cannot be said to have established anything.

Whatever meaning the author intended, he should have realized the need to clearly communicate it to the reader. Since all of my guesses as to the meaning of (1.4) have led to inconsistencies, in the absence of a better guess I have to consider (1.4) as too vague to be meaningful, in Pauli's famous words, "not even wrong". The whole book is like that.

To finish up, the $\vec{k} \cdot \vec{dl}$ above is engineering notation for a 1-form on three-dimensional space. The text claims (effectively, not in so many words) that $\vec{k} \cdot \vec{dl}$ is a 1-form on 3-space which has been naturally obtained from quantum mechanics and which satisfies its key equation (1.4) above. In classical electrodynamics, the three-potential $\vec{A}$ (here $\vec{A}$ is the space part of the four-potential A discussed above) has the property that its integral around a closed loop, $\oint \vec{A} \cdot \vec{dl}$, gives the magnetic flux threading that loop. For a superconducting loop, that magnetic flux is an integral multiple of a constant, similar to the text's claim mentioned above that $\oint \vec{k} \cdot \vec{dl} = 2\pi n$. From this the text concludes (in different language) that the supposedly naturally occurring 1-form $\vec{k} \cdot \vec{dl}$ must be a constant multiple of the three-potential 1-form $\vec{A} \cdot \vec{dl}$ of classical electrodynamics. (Note that this conclusion is actually an *assumption.*) Finally, by handwaving analogies the text promotes the 1-form $\vec{A} \cdot \vec{dl}$ on 3-space to a 1-form A on four-dimensional Minkowski space and eventually obtains standard classical electrodynamics in the usual way outlined above.

Is there anything of interest in the book? Well, some may find of interest an

[1] In case this seems puzzling, recall that a 1-form f at point $\vec{x} \in R^3$ is a linear mapping which assigns to each vector $\vec{v}$ in the tangent space at $\vec{x}$ (in engineering language, $\vec{v}$ is a "bound" vector with "tail" at $\vec{x}$), a real number $f(\vec{v})$. The 1-form which I am denoting (to remain as close as possible to the text's notation) $\vec{k} \cdot \vec{dl}$ assigns to a vector $\vec{v}$ as above, the number $\vec{k} \cdot \vec{v}$. When such a 1-form is integrated over a curve C, the result is, in the notation of vector calculus, $\int_C \vec{k} \cdot \vec{dl}$. If C happens to be a closed curve, this result is commonly denoted $\oint \vec{k} \cdot \vec{dl}$, as in the text.

11-page "Personal Preface" describing, among other things, the author's relationship with and impressions of Richard Feynman. Mead was an undergraduate student of Feynman and later his colleague at Caltech.

I have mixed feelings about these. His reminiscences sound sincere, but also seem to me to have a flavor of name-dropping. For example, he discusses a "sticking point" in his development of electrodynamics which held him up for years, and informs us that "it is resolved in this treatment in a way that Feynman would have liked". It seems presumptuous to claim to know what a great, deceased physicist would have thought about this work.

I suggest that anyone contemplating buying this book first check it out of a library and try to read the pages quoted above (pp. 9-14) in detail. If they seem clear to you in the light of the above analysis, then you can reasonably conclude that I didn't understand the author's arguments and discount this review.

- July 23, 2006: This is the second posted version of this file. The first version, posted yesterday, July 22, 2006, contained some minor errors, which are corrected in this version. The errors did not affect the conclusions.

- July 26, 2006: A minor typo ("magnetic field" changed to "magnetic flux") was corrected.

- October 25, 2006: I just discovered that the text of Part 1 of the book seems nearly identical to the author's paper

  > Mead, C. A. , Collective Electrodynamics I, *Proc. Natl. Acad. Sci. USA* (PNAS) **94** (1997), pp. 6013-6018.

  In particular, the book's first few pages from which most of the review's quotes are drawn also appear seemingly verbatim in the paper. (I haven't performed a word by word comparison, but they look identical.)

  The book's "Personal Preface" states that Part 1 of the book is "an expanded version" of the above paper, but I haven't noticed any text in Part 1 which doesn't appear verbatim in the paper. The main difference seems to be that the book contains some helpful drawings (mainly representations of superconducting coils) which the paper lacks. This paper is available in full on the PNAS web site www.pnas.org (search for Mead, Carver), so anyone interested in evaluating the review's analysis can obtain the original text from this source without obtaining the book.

  The paper states that it "is the first in a series", as its title would suggest. My search engines have not located subsequent published papers in this intended series.