Introduction to Probability Models

`Offered in Fall 2013 as MATH 597, Tu-Th 5:30-6:45`
Instructor: Mirjana Vuletić `mirjana.vuletic@umb.edu` 617-287-7418

Brief Description

Topics covered: discrete Markov chains, continuous-time Markov chains, Poisson processes, renewal theory, and martingales. Optional topics: queuing theory, reliability theory, Brownian motion, and random variables simulation. Applications to biology, physics, computer science, economy, and engineering will be presented.

Prerequisites

Math 345 or permission of the instructor. In addition to having a solid foundation in probability (Math 345 or its equivalent), students taking this course will need to be able to perform computations using calculus and linear algebra.

Course Summary

This is a first course on probability models with a strong emphasis on stochastic processes. Stochastic processes are used as models for many real-world phenomena where one needs to take into account the possibility of randomness. The course covers a variety of models, their properties and applications. The general list of topics is given in the Brief Description, where the particular choice of optional topics will be chosen based on audience backgrounds and interests.

The course aim is to enable students to approach real-world phenomena probabilistically and build effective models. As such this course may appeal to mathematically inclined students in fields such as biology, physics, computer science, engineering and economics as well as mathematics students who want to learn more about applications of probability. The course emphasizes models and their applications over the rigorous theoretical framework behind them, yet critical theory that is important for understanding the material is also covered.

The difficulty of the course is of an upper level undergraduate or an introductory graduate course.

The course begins with a review of probability. The major topics included are the following:

- *Discrete Markov chains* These illustrate one of the simplest stochastic processes, but with a wide range of applications. A discrete Markov chain is a mathematical system that at different discrete times can be in one of several possible states. It possesses the Markov property that given the present state, the future is independent of the past. The system can transition from one state to another according to transition probabilities. Various concepts related to Markov chains will be presented, including reducibility, periodicity and recurrence. A *branching process* will be presented as an example of the discrete Markov chain. *Markov chain Monte Carlo methods* will be introduced.

- A *continuous-time Markov chain* is the continuous time analog of a discrete Markov chain that also satisfies the Markov property. Birth-death processes will be presented as examples of continuous-time Markov chains. The concept of time reversibility will be discussed.

- *Poisson processes* and *renewal processes* are also examples of continuous time Markov-chains. A Poisson process can be viewed as a counting process where the times between successive events are independent and exponentially distributed with the same mean. A renewal process is a generalization of the Poisson process for arbitrary holding times. More precisely, this is a counting process for which the times between successive events are independent and equally distributed with an arbitrary distribution. Properties and applications of Poisson and renewal processes will be presented.

- *Martingales* are a model of a fair game where knowledge of past events does not help predict future winnings. More precisely, the expected value in the future is equal to the present observed value. An unbiased random walk and *gambler's fortune* are examples of martingales. Martingale stopping time theorem will be presented.

Optional topics include the following:

- *Queuing Theory* deals with waiting line theory where customers arrive in some random manner. In queuing theory one is interested in limiting probabilities or quantities like the average number of customers in the waiting line or the average time a customer spends in the waiting line. Some special queuing models can be discussed.

- *Reliability Theory* is concerned with determining the probability that a system will function. The system can consist of many components, mutually connected in different ways, e.g. a series system or a parallel system, where each component has a certain probability of working. The probability that the system will function depends on the probabilities that components work and the way components are connected. In reliability theory one is interested in a quantity like the mean lifetime of the system.

- *Brownian Motion* is the motion exhibited by a small particle that is totally immersed in a liquid or gas, where the particle moves as a result of being bombarded by the fast-moving atoms or molecules in the gas or liquid. Since its discovery in 1827, it has been used in areas such as statistical testing of goodness of fit, analyzing the price levels of the stock market, and quantum mechanics.

- *Random Sampling Techniques*
  An important problem in applied probability and computational statistics is how to simulate random variables. Various *random sampling techniques* can be discussed: pseudo-random generators, the inverse transforming method, the rejection method and Monte Carlo simulations. The latter are especially useful for simulating phenomena with significant uncertainty in inputs and systems with large degrees of freedom. They have various applications in physics, engineering, biology and statistics.

Course Books

The textbook is:

1. *Introduction to Probability Models*, Sheldon M. Ross, Academic Press; 10th edition (December 17, 2009), 800 pages

Other relevant books covering the course topics:

2. *Probability and Random Processes*, Geoffrey R. Grimmett and David R. Stirzaker, Oxford University Press, USA; 3rd edition (August 2, 2001), 608 pages

3. *Adventures in Stochastic Processes*, Sidney I. Resnick, Birkhauser ; 1th edition (September, 1992), 638 pages

4. *Introduction to Stochastic Processes*, Gregory F. Lawler, Chapman and Hall/CRC; 2nd edition (May 16, 2006), 248 pages

5. *Stochastic Processes*, Sheldon M. Ross, Wiley; 2nd edition (January 1995), 510 pages