Title: Spectral Sequences: some representation-theoretic applications

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ABSTRACT.

Spectral sequences were introduced by Jean Leray, in 1946, in connection with problems in algebraic topology where a tool more powerful than exact sequences was required to relate, for example, various cohomology or homotopy groups. Their applications abound in many areas that range, for example, from algebraic geometry to BRST cohomology of 2-dimensional gravity (coupled to matter). We present some expository remarks on spectral sequences with some concrete examples and applications in representation theory. For example, we consider (i) nil-radical Lie algebra cohomology with coefficients in the space of K-finite vectors of an irreducible unitary representation of a semisimple Lie group G with maximal compact subgroup K. The results here lead, for example, to (automorphic) multiplicity formulas for derived functor modules (representations constructed by the method of cohomological parabolic induction), and in particular to a result conjectured by R.Langlands (in the discrete series case) and more generally to a solution of G.Warner's 3rd problem (for representations of holomorphic type)—without recourse to the trace formula (ii) a BGG type resolution of holomorphic Verma modules that extends initial work of R.Stanke—i.e. a result for a general Hermitian G/K.