Graphing Practice

Math 130 Kovitz

For each equation, draw a rough graph. You will need to find and state:

- The quadrants in which the graph lies. The domain and range.
- The endpoints of the graph, if there are any.
- The maximum or minimum value or values, and for which \( x \) they occur.
- The exact coordinates of several—say six or seven—points on the graph.
- Any symmetries. Draw that line with its equation; or plot that point with its coordinates. They might not be any of our usual four cases.
- Whether the graph is connected (continuous), and its shape. (To find the shape, you might use algebra to find an alternate form of the equation.)
- Where the graph is increasing or decreasing, and its concavity.
- The exact coordinates of all intercepts. Then plot and label those intercepts.
- The equations of any asymptotes.

1. \( xy^2 = 36 \)
2. \( y = -|x + 7| + 4 \)
3. \( y^2 = 5 - x^2 \)
4. \( 12 = \sqrt{x} \sqrt{y} \)

ANSWERS follow.
ANSWERS

1. • The quadrants are the first and the fourth. The domain is all positive reals, and the range is all real numbers except 0.
   • This graph has no endpoints.
   • There is neither a maximum or minimum value.
   • Points on the graph: \((1, 6), (1, -6), (4, 3), (4, -3), (9, 2), (9, -2)\).
   • The line of symmetry is the \(x\)-axis.
   • The graph consists of two connected curves of hyperbolic shape; and it is situated fairly close to the relevant axes. Those axes are asymptotes.
   • The graph is increasing in the fourth quadrant and decreasing in the first. It is concave up in the first quadrant and concave down in the fourth.
   • There are no intercepts.
   • The \(y\)-axis: \(x = 0\); and the positive \(x\)-axis: \(y = 0\) and \(x > 0\).

2. • The quadrants are the 2nd, 3rd, and 4th. The domain is all reals, and the range is all reals less than or equal to 4.
   • This graph has no endpoints.
   • The maximum value is 4, and it occurs when \(x = -7\).
   • Other points on the graph:
     \((-20, -9), (-5, 2), (-4, 1), (-2, -1), \left(\frac{1}{7}, -3\frac{1}{7}\right), (5, -8)\).
   • The line of symmetry is \(x = -7\).
   • The graph is two connected rays, an increasing ray for \(x > -7\), and a decreasing ray for \(x > -7\).
   • The graph is increasing for \(x > -7\) and decreasing for \(x < -7\).
   • The intercepts are: \((-11, 0), (-3, 0)\), and \((0, -3)\).
   • There are no asymptotes.

3. • The graph lies in all four quadrants. The domain and the range are both \([-\sqrt{5}, \sqrt{5}]\).
   • This graph has no endpoints.
   • The maximum and minimum values are \(\pm \sqrt{5}\); they occur when \(x = 0\).
   • Points on the graph:
     \((-\sqrt{5}, 0), (-\sqrt{3}, -\sqrt{2}), (-2, 1), (1, 2), (\frac{1}{2}\sqrt{10}, \frac{1}{2}\sqrt{10}), (2, -1)\).
   • All four usual symmetries are present.
   • The graph is a circle with radius \(\sqrt{5}\), centered at the origin. All circles have connected graphs.
   • The graph is increasing in quadrants 2 and 4, and decreasing in quadrants 1 and 3. It is concave up in the third and fourth quadrants and concave down in the first and second quadrants.
   • Intercepts; \((0, \pm \sqrt{5}), (\pm \sqrt{5}, 0)\).
   • There are no asymptotes.
4. • The graph lies in the first quadrant only. The domain and range are each the positive real numbers.
  • There is no endpoint.
  • There is no maximum or minimum value.
  • Points on the graph: (2, 72), (4, 36), (8, 18), (16, 9), (12, 12), (144, 1).
  • There are no symmetries.
  • The graph is a single connected curve of hyperbolic shape; and the positive axes are asymptotes.
  • The graph is decreasing and concave up.
  • There are no intercepts.
  • The asymptotes are the positive x- and y-axes.