Notes on the Unit Circle
Math 130 Kovitz

The Unit Circle
The equation $x^2 + y^2 = 1$ produces a graph which is a circle with center at the point $(0, 0)$ and radius 1.

Proof: Let $(a, b)$ be an ordered pair in the solution set of $x^2 + y^2 = 1$. That means that $a^2 + b^2 = 1$. Draw the right triangle obtained when the point $(a, b)$ is plotted in the plane. By the Pythagorean theorem $a^2 + b^2 = \text{hypotenuse}^2$. That means that the hypotenuse of this right triangle is 1. From this we conclude that the distance of the point $(a, b)$ from the point $(0, 0)$ is 1 because that distance is exactly the hypotenuse. We have thus shown that an arbitrary point in the solution set of $x^2 + y^2 = 1$ is on the circle with center at $(0, 0)$ and radius 1.

Domain and Range
The domain of the unit circle (the set of all $x$-coordinates of points on the circle) is: $\{x | -1 \leq x \leq 1\}$.

The range of the unit circle (the set of all $y$-coordinates of points on the circle) is: $\{y | -1 \leq y \leq 1\}$.

Finding Points on the Unit Circle
Here are some examples of finding points on the unit circle.

To find the four intercepts $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$ we let $x = 1$, $y = 1$, $x = -1$, or $y = -1$. For example when $x = 1$, we have $1^2 + y^2 = 1$, $y^2 = 0$, and $y = 0$. From this we conclude that $(1, 0)$ is the only point on the unit circle with $x = 1$. Similar methods will produce the other three intercepts.

When $x = .8$, we have $.8^2 + y^2 = 1$, yielding $.64 + y^2 = 1$, $y^2 = .36$, and $y = \pm .6$. The points on the unit circle when $x = .8$ are thus $(.8, .6)$ and $(.8, -.6)$.

It is easy to see that when $x = -0.8$ the points on the unit circle are $(-.8, .6)$ and $(-.8, -.6)$.

When $x = .5$, we have $.5^2 + y^2 = 1$, yielding $.25 + y^2 = 1$, $y^2 = .25 = 3/4$, and $y = \pm \sqrt{3/4} = \pm \sqrt{3}/2 = \pm 1.732/2 = \pm .866$. The points on the unit circle when $x = .5$ are thus approximately $(.5, .866)$ and $(.5, -.866)$.

It is easy to see that when $x = -0.5$ the points on the unit circle are approximately $(-.5, .866)$ and $(-.5, -.866)$.

When $x = y$, substituting $x$ for $y$ in the equation $x^2 + x^2 = 1$ yields $2x^2 = 1$, $x^2 = 1/2$, and $x = \pm \sqrt{1/2} = \pm 1/\sqrt{2} = \pm (\frac{\sqrt{2}}{2}) = \pm \frac{\sqrt{2}}{2} \approx \pm \frac{1.414}{2} = \pm .707$. We also have $y = x$. This means that the points on the unit circle when $x = y$ are approximately $(.707, .707)$ and $(-.707, -.707)$.

The Graph of the Unit Circle
Here is the graph of the unit circle, with the points found so far labeled with their coordinates.