More Even and Odd Function Practice
Math 130 Kovitz

In problems 1. through 11.: Decide whether the function $f$ with the given rule is even, odd, or neither. Justify your answer.

1. $f(x) = 1/x$
2. $f(x) = (x^2 + 4)(x - 2)(x + 2)$
3. $f(x) = \begin{cases} 5x + 4 & \text{if } x > 0 \\ 5x - 4 & \text{if } x < 0 \end{cases}$
4. $f(x) = \frac{1-x}{1+x} + \frac{1+x}{1-x}$.
5. $f(x) = \frac{1-x}{1+x} - \frac{1+x}{1-x}$.
6. $f(x) = \frac{x-1}{x}$.
7. $f(x) = x - \frac{1}{x}$
8. $f(x) = |x|$
9. $f(x) = \sqrt{|x|}$
10. $f(x) = \frac{x^2 - 4x + 4}{x}$
11. $f(x) = |x|/x$
12. $f(x) = \frac{x^3 - 1}{x - 1}$.

For each of the following problems, decide whether the solutions to the equation constitute an odd function, an even function, neither, or both.

13. $x^4 = y^4$
14. $x^2 + y^2 = 0$, considering the solutions over the real numbers only.
15. $x^2 + y^2 = 1$ with $y \geq 0$.

Answers below
Answers with Justifications

1. Odd. For all \( a \): \( f(-a) = 1/(-a) \) and \( -f(a) = -1/a \). They are equal.

2. Even. It reduces to \( x^4 + 16 \), which is even by the rule of even powers.

3. Odd. If \( a > 0 \): \( f(a) = 5a + 4 \) and \( f(-a) = 5(-a) - 4 = -5a - 4 = -(5a + 4) = -f(a) \). If \( a < 0 \): \( f(a) = 5a - 4 \) and \( f(-a) = 5(-a) + 4 = -5a + 4 = -(5a - 4) = -f(a) \).

   It is much easier to look at the graph and note that it is symmetric through the origin.

4. Even. It simplifies to \( f(x) = \frac{2(1 + x^2)}{1 - x^2} \), so it’s even by the rule of even powers.

5. Odd. It simplifies to \( f(x) = \frac{-4x}{1 - x^2} \), so \( f(a) = \frac{-4a}{1 - a^2} \) and \( f(-a) = \frac{4a}{1 - a^2} = -f(a) \).

6. Neither. Because \( f(1) = 0 \) and \( f(-1) = 2 \), it cannot possibly be odd or even.

7. Odd. \( f(-a) = -a + \frac{1}{a} = -\left(a - \frac{1}{a}\right) \).

8. Even. \( f(-a) = |a| = |-1||a| = |a| \).

9. Even. \( f(-a) = \sqrt{|-a|} = \sqrt{-1||a|} = \sqrt{|a|} \).

10. Neither. \( f(2) = 0 \) but \( f(-2) = -8 \). Simplifying the numerator to \( (x - 2)^2 \) does not change this fact.

11. Odd. \( f(-a) = |a|/(-a) = -|a|/a = -(|a|/a) = -f(a) \).

12. Neither. \( f(2) = 7 \) but \( f(-2) = 3 \).

   No need to simplify as \( (x - 1)(x^2 + x + 1)/(x - 1) = x^2 + x + 1 \), but that also would be ‘neither’ from \( f(2) \) and \( f(-2) \).

13. Neither. It is not a function, because both \( (2, -2) \) and \( (2, 2) \) are solutions, and the graph violates the vertical line test.

14. Both. There is only one point \((0,0)\). So the domain is \( \{0\} \) (just \( x = 0 \)).

   Conclude that both \( f(-0) = f(0) = 0 \) and \( f(-0) = -f(0) = -0 \) hold.

15. Even. It is a function because \( f(x) = \sqrt{1-x^2} \) is a valid formula. Dispense with the \( \pm \) of the solution once \( y = f(x) \) is known to be non-negative. For each \( x \) in the domain, there will be exactly one \( y \); the positive one. From that it is clear that the graph will pass the vertical line test. Had the \( y \)'s not been stipulated to be positive, the equation would not be the equation of a function.

   If \((a,b)\) is a solution, then \((-a,b)\) will also be a solution. That's all one needs to show the function is even.

   The procedure of separating a circle into two semicircles is sometimes necessary to graph a circle on a graphing calculator. It is also used in higher mathematics when functions are needed and the relation at hand is the circle.