The Difference Quotient, and the slope of the tangent line to a curve. Name ___________ Section ____

Objectives: (1) to understand and calculate the difference quotient.

(2) to use the difference quotient to find the slope of the tangent line to a function curve.

Let f(x) be a function.

A secant line to f(x) is a line passing through two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ of f(x).

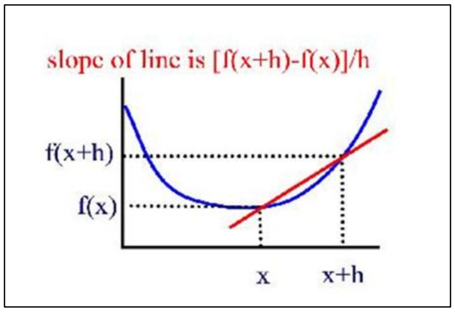
The slope of the secant line is $\Delta y/\Delta x$, or: $[f(x_2) - f(x_1)] / (x_2-x_1)$.

We want to keep x_1 fixed, and squeeze x_2 closer and closer to x_1.

To emphasize this idea, we rewrite the slope of the secant line substituting x for x_1, and (x+h) for x_2. See the picture at the right.

Since (x+h) – (x) = h, the denominator in the slope expression is h. The expression is called the **difference quotient**, and is written:

[f(x+h) – f(x)]/ h .



Examples.

Example 1. f(x) = 3x. This is a linear equation. Given any two points on this line, we know the slope is 3. Calculating using the difference quotient: slope = [f(x+h) – f(x)]/ h = [3(x+h) -3x] / h = [3x+3h-3x]/h =3h/h = 3.

Example 2. $f(x) = 4x^2$. This is a quadratic equation. The slope of the secant line depends on the point x, and also on the distance h between x and x_2. (see the picture above: the red line is the secant line). Calculating using the difference quotient, slope = = $[f(x+h) – f(x)]/ h = [4(x+h)^2 -4x^2] / h =$ $[4(x^2+2xh+h^2) -4x^2]/h = [4x^2+8xh+4h^2-4x^2]/h = [8xh+4h^2]/h = [h(8x+4h)]/h = (h/h) (8x+4h) = 1 (8x+4h) =$ 8x+4h.

What we are really interested in is the slope of the secant line when the two points are very, very close. We can't set the value of h to zero, because then the difference quotient becomes (0/0) which is undefined. However, we do a limiting process. We let h get closer and closer to zero, and try to find out how the slope of the secant line changes. **If the slope of the secant line has a limiting value**, then that value will approach the slope of the **tangent line** (the line that passes through (x,f(x)) and which has the same direction as the function curve at that point).

Back to example 2. As the value of h approaches zero, then the value of the slope of the secant line approaches 8x, since 4h gets very small. So the tangent line to the parabola $f(x)=4x^2$ at the point (x,x^2) has slope m= 8x. For example, at (x,y)= (-2,16), then the slope of our parabola is 8(-2) = -16 at (-2,16). This value, the slope of the tangent line to a function y = f(x), at the point (x, f(x)), is called **Df(x)**, or **the derivative of f(x) at x**.

The derivative is calculated in two steps:

Step 1. Find the difference quotient.

Step 2. Find the limit, if it exists, of the difference quotient, as h approaches zero. (symbol: $h \to 0$).

This limit process is a little bit tricky, and requires some mathematical machinery. Right now, we want to calculate difference quotients, and use some common sense to evaluate the limit value.

Problems. Find the difference quotient, and, if possible, the derivative, of each function:

1. $f(x) = 3x^2$
2. f(x) = 2x-5
3. $f(x) = e^x$
4. $f(x) = x^2-2x+1$
5. $f(x) = x^3$
6. $f(x) = (1/2) x^2 - 3$
7. f(x) = cos(x). You will need the cos(x+y) formula. Do not try to calculate the limit when $h \to 0$.
8. F(x) = sin(x). You will need the sin(x+y) formula. Do not try to calculate the limit when $h \to 0$.