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A **quadratic function** is a function of the form $\mathbf{y = ax^2+bx+c}$, where a, b, and c are real numbers.

Skill #1. Evaluating a **quadratic function**.

Example. $f(x) = 3x^2-4x-5$. Evaluate f(x) for x= -2/3.

Answer: $f(-2/3) = 3(-2/3)^2-4(-2/3)-5 = 3(4/9) -4(-2/3) -5 = 4/3 + 8/3 - 15/3 = -3/3 = -1$.

Skill #2. Calculating the **discriminant**.

A quadratic function has a **discriminant** $\boldsymbol{\Delta = b^2-4ac}$.

The discriminant of the function $f(x) = 3x^2-4x-5$ is calculated as follows.

First write down a,b, and c:

a = 3
b = -4
c = -5

Then write:

$\Delta = b^2-4ac$

$= (\ \ )^2 - 4 (\ ) (\ )$

$= (-4)^2 - 4 (3) (- 5)$

$= 16 + 60 = 76.$

Skill #3. Using the **discriminant**.

The **discriminant Δ** tells you several things.

(a) If Δ is a perfect square integer (Δ = 0,1,4,9,16,25, etc), then the quadratic polynomial factors over the integers.

(b) If Δ > 0, then the quadratic function has two different real roots.

(c) If Δ = 0, then the quadratic function has one real double root.

(d) If Δ < 0, then the quadratic function has no real roots, but it has two complex conjugate roots.

Example: the polynomial function $f(x) = 3x^2-4x-5$ has discriminant 76. Since 76 is not a perfect square, the polynomial cannot be factored over the integers. Since 76 > 0, the polynomial has two different real roots.

Skill #4. Does the parabola open up or down?

The graph of a quadratic function opens upwards (∪) if the coefficient of x^2 is a positive number. The graph opens downwards (∩) if the coefficient of x^2 is a negative number.

Skill #5. Finding the x-coordinate of the vertex, and the **line of symmetry (axis of symmetry**).

The **vertex of a parabola** (or quadratic function) is the (x,y) point which is the highest or lowest point on the curve. The **x-value of the vertex** is half-way between the two roots, or the average of the two roots. It may be calculated directly by the formula **x = -b/(2a)** .

Example: the x-value of the vertex of the equation $f(x) = 3x^2-4x-5$ is

x= - b/(2a) = - (-4)/(2(3)) = 4/6 = 2/3.

The **line of symmetry** (**axis of symmetry**) of the parabola is the **vertical line through the vertex**.

Skill #6. Finding the **y-coordinate of the vertex**.

The y-coordinate of the vertex is found by evaluating f(x) at the x-coordinate of the vertex. In the above example, the y-coordinate of the vertex is f(-b/(2a)) = f(2/3) = -19/3. (see Skill #1 above).

Skill #7. Finding the **roots** (also called the **x-intercepts**).

The **roots** of a function are the **x-values for which the function value (y-value) is zero**. Use factoring; but if the polynomial is not factorable, use the quadratic formula.
The QF (quadratic formula) for the roots of a quadratic function may be stated as follows:

If $ax^2+bx+c = 0$, then .

Before evaluating the QF, be sure to write the values of a,b,c (see skill #2).
Note: from this formula, you can see that -b/(2a) is the average of the roots. Further, the distance between the roots is ± √Δ / a,   and the distance along the x axis from either root to the line of symmetry is   (√Δ) / (2a) .

Skill #8. Finding the **y-intercept**.

Find the y-intercept by evaluating the polynomial at x=0.
In our example $f(0) = 3(0)^2 -4(0) -5 = -5$. The y-intercept is at the point (0,-5).

Skill #9. Finding the symmetric point.

The symmetric point is at the same height as the y-intercept point. It is symmetric about the axis of symmetry to the y-intercept; so it is the same distance from the axis of symmetry as the y-intercept: (y-intercept) --------- axis ------------ (symmetric point).
So the symmetric point is a distance   2 (-b/2a) = -b/a from the y-axis.

Skill #10. Graphing the quadratic function.

When graphing, do all the above things in order.
Your sketch should show: (a) the roots; (b) the vertex ; (c) the line of symmetry; (d) the y-intercept; (e) the symmetric point; and perhaps one or two other points.

Skill #11. Solving the quadratic by completing the square. [not discussed here].

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Exercises. Graph the following quadratic functions. Calculate the roots, the vertex, and the discriminant. Find the y-intercept and the symmetric point. Show at least one or two other points on the graph.

	Equation	a	b	c	Δ	# of real roots	vertex	y-intercept	Symmetric point	roots
1	$f(x) = x^2-6x+8$									
2	$f(x) = x^2+4x+3$									
3	$f(x) = x^2-2x-15$									
4	$f(x) = x^2+2x-8$									
5	$f(x) = -x^2-2x+3$									
6	$f(x) = -x^2-4x+5$									
7	$f(x) = 2x^2+7x+3$									
8	$f(x) = 3x^2-7x+2$									
9	$f(x) = -4x^2 +4x-1$									
10	$f(x) = x^2+6x+9$									
11	$f(x) = x^2+2x+5$									
12	$f(x) = -2x^2+4x-3$									
13	$f(x) = x^2+2x-5$									
14	$f(x) = -x^2+4x-1$									

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	Equation	a	b	c	Δ	# of real roots	vertex	y-intercept pt	Symmetric point	roots
1	$f(x) = x^2-6x+8$	1	-6	8	4	2 (factors)	(3,-1)	8	(6,8)	2,4
2	$f(x) = x^2+4x+3$	1	4	3	4	2 (factors)	(-2,-1)	3	(-4,3)	-1,-3
3	$f(x) = x^2-2x-15$	1	-2	-15	64	2 (factors)	(1,-16)	-15	(2,-15)	5,-3
4	$f(x) = x^2+2x-8$	1	2	-8	36	2 (factors)	(-1,-9)	-8	(-2,-8)	2,-4
5	$f(x) = -x^2-2x+3$	-1	-2	3	16	2 (factors)	(-1,4)	3	(-2,3)	-3,1
6	$f(x) = -x^2-4x+5$	-1	-4	5	36	2 (factors)	(-2,9)	5	(-4,5)	-5,1
7	$f(x) = 2x^2+7x+3$	2	7	3	25	2 (factors)	(-7/4, -25/8)	3	(-7/2,3)	-3, - ½
8	$f(x) = 3x^2-7x+2$	3	-7	2	25	2 (factors)	(7/6, -25/12)	2	(7/3,2)	2, 1/3
9	$f(x) = -4x^2 +4x-1$	-4	4	-1	0	2	(1/2,0)	-1	(1,-1)	½ , ½
10	$f(x) = x^2+6x+9$	1	6	9	0	1 double	(-3,0)	9	(-6,9)	-3,-3
11	$f(x) = x^2+2x+5$	1	2	5	-16	0	(-1,4)	5	(-2,5)	$-1 \pm 2i$
12	$f(x) = -2x^2+4x-3$	-2	4	-3	-8	0	(1,-1)	-3	(2,-3)	$-1 \pm \sqrt{2}\, i$
13	$f(x) = x^2+2x-5$	1	2	-5	24	2	(-1,-6)	-5	(-2,-5)	$-1\pm\sqrt{6}$
14	$f(x) = -x^2+4x-1$	-1	4	-1	12	2	(2,3)	-1	(4,-1)	$-2 \pm\sqrt{3}$