In the following exercise you will practice drawing linear graphs.

EXERCISE 3-2

Do These Quickly

The following problems are intended to refresh your skills from the first two chapters and from previous courses. You should be able to do all 10 in less than 5 minutes.

Q1. Write a rational number that is not an integer.

Q2. Write an integer that is not positive.

Q3. Write an odd prime number.

Q4. Solve: $3x + 7 = 31$

Q5. Evaluate $5x - 2$ if x is 3.

Q6. Find 20% of 63.

Q7. What axiom is illustrated: "If $x = y$, then $y = x$."

Q8. Evaluate $|2 - 5x|$ if x is 3.

Q9. Sketch the graph of a relation that is not a function.

Q10. Add $\frac{2}{3}$ and $\frac{3}{4}$.

Work these problems.

For Problems 1 through 20, plot the graph neatly on graph paper. Use the slope and y-intercept, where possible.

1. $y = \frac{3}{5}x + 3$
2. $y = \frac{5}{2}x - 1$
3. $y = -\frac{3}{2}x - 4$
4. $y = -\frac{1}{4}x + 3$
5. $y = 2x - 5$
6. $y = 3x - 2$
7. $y = -3x + 1$
8. $y = -2x + 6$
9. $7x + 2y = 10$
10. $3x + 5y = 10$
11. $x - 4y = 12$
12. $2x - 5y = 15$
13. $y = 3x$
14. $y = -2x$
15. $y = 3$
16. $y = -5$

17. $x = -4$

18. $x = 2$

19. $y = 0$

20. $x = 0$

21. Relations such as in Problems 15 through 20 where

$$x = \text{constant} \quad \text{or} \quad y = \text{constant}$$

are not called linear functions, even though their graphs are straight lines. However, the reason is different in each case. Explain why such relations are not called linear functions.

22. *Intercepts Problem* In the definition of intercepts, it says *the* y-intercept, but *an* x-intercept. Sketch a graph which shows that a function could have more than one x-intercept. Explain why a function could not have more than one y-intercept.

23. *Division by Zero Problem* Evaluate $\frac{1}{0.1}$, $\frac{1}{0.01}$, $\frac{1}{0.001}$, and $\frac{1}{0.0001}$. What happens to the size of a fraction as its denominator gets very close to zero? Why would $\frac{1}{0}$ be larger than any real number? What name is used for a quantity that is larger than any real number?

24. *Slope Proof Problem* Prove that if $y = mx + b$, where m and b are constants, then m is the slope. Write a reason to justify each step in the proof.

25. *Another Form of the Linear Function Equation*
    a. Show that the relation

    $$y - 4 = 2(x - 5)$$

    is a *linear* function by transforming to $y = mx + b$. $y = 2x - 6$
    b. Plot the graph of the function. Check student work.
    c. What does the number 2 in the original equation tell you about the graph? The slope is 2; the graph slopes up.
    d. The coordinates of a point on the graph are concealed in the original equation! What point? (5, 4)

3-3 OTHER FORMS OF THE LINEAR FUNCTION EQUATION

Suppose that a relation has the equation

$$y - 4 = 2(x - 5).$$

If you substitute (5, 4) for (x, y), both members of the equation are zero. So (5, 4) is a point on the graph because it makes the equation a *true* statement. By distributing the 2, then adding 4 to each member, the equation becomes

$$y = 2x - 6.$$

So the relation is a *linear function* with slope 2. You may already have discovered this if you worked Problem 25 in Exercise 3-2.

An equation such as $y - 4 = 2(x - 5)$ is said to be in *point-slope* form because the coordinates of a point and the slope of the line appear in the equation. The familiar $y = mx + b$ form of the linear function equation is called *slope-intercept* form.

A linear function equation can be written as $3x + 4y = 13$. In this text, the name "$Ax + By = C$ form" is used if both variables are on one side, and the constant is on the other.

FORMS OF THE LINEAR FUNCTION GENERAL EQUATION

$y = mx + b$	Slope-intercept form.
$y - y_1 = m(x - x_1)$	Point-slope form. (x_1, y_1) is a point on the graph.
$Ax + By = C$	"$Ax + By = C$" form. A, B, and C stand for constants.

Objective:
Given an equation in point-slope form, plot the graph quickly, and transform it to the other two forms.

EXAMPLE

If $y - 5 = -\dfrac{3}{2}(x + 1)$,

a. Plot the graph quickly.
b. Transform the equation to slope-intercept form.
c. Transform the equation to $Ax + By = C$ form, where A, B, and C stand for *integer* constants.

Solution
a. From the equation, the slope is $-\frac{3}{2}$. A point on the graph is $(-1, 5)$ because substituting 5 for y makes the left member 0 and substituting -1 for x makes the right member 0. The graph is shown in Figure 3-3.

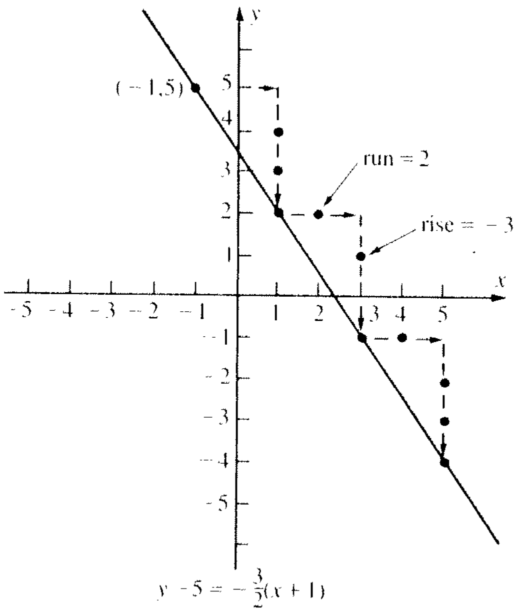


Figure 3-3

b. The equation can be transformed to slope-intercept form by distributi the $-\frac{3}{2}$, then adding 5 to each member.

$$y - 5 = -\frac{3}{2}(x + 1),$$

$$y - 5 = -1.5x - 1.5$$

$$\underline{\underline{y = -1.5x + 3.5}}$$

c. Starting with the answer in part (b), you can add $1.5x$ to each memb then multiply by 2 to make each coefficient an integer.

$$1.5x + y = 3.5$$

$$\underline{\underline{3x + 2y = 7}}$$

The following exercise gives you practice using the point-slope form to plot graphs, and transforming from point-slope form to the other forms.

EXERCISE 3-3

Do These Quickly

The following problems are intended to refresh your skills. You should be able to do all 10 in less than 5 minutes.

Q1. Sketch a graph in which y increases as x increases.

Q2. Simplify: $3 + 2(x - 5)$

Q3. Multiply: $(2x - 7)(x + 3)$

Q4. Solve: $2x - 7 = 31$

Q5. Evaluate $5x^2$ if x is 3.

Q6. What percent of 40 is 12?

Q7. What axiom is illustrated: "$x = x$."

Q8. Evaluate: $\sqrt{49}$

Q9. Multiply $\frac{2}{3}$ by $\frac{3}{4}$.

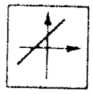
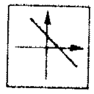
Q10. Factor: $x^2 + 4x - 5$

Work these problems.

For Problems 1 through 10,

a. Plot the graph, showing clearly the point and slope that appear in the equation.
b. Transform the equation to slope-intercept form.
c. Transform the equation to $Ax + By = C$ form, where A, B, and C are all integers.

1. $y - 2 = \frac{3}{5}(x - 1)$
2. $y - 3 = \frac{2}{5}(x - 6)$
3. $y + 4 = \frac{7}{2}(x - 3)$
4. $y + 1 = \frac{7}{3}(x - 4)$
5. $y - 6 = -\frac{1}{4}(x + 2)$
6. $y - 2 = -\frac{1}{2}(x + 5)$
7. $y + 1 = -2(x + 4)$
8. $y + 6 = -3(x + 2)$
9. $y = \frac{1}{3}(x - 12)$
10. $y - 5 = \frac{2}{5}x$

If you really understand a concept, you should be able to use it *backward* as well as forward. For Problems 11 through 14, write an equation in point-slope form for the linear function described.

11. Contains the point $(5, 7)$, and has slope -3.
12. Contains the point $(6, 3)$, and has slope 5.
13. Contains the point $(-2, 5)$, and has slope $\frac{9}{13}$.
14. Contains the point $(7, -9)$, and has slope $-\frac{22}{7}$.

3-4 EQUATIONS OF LINEAR FUNCTIONS FROM THEIR GRAPHS

Suppose someone says, "If the equation is $y = 3x - 8$, what are the slope and y-intercept?" You would say, "That's easy! They are 3 and -8." It is just as easy for you to answer the question, "If the slope and y-intercept are -5 and 13, what is the equation?" The answer is

$$y = -5x + 13.$$

In this section you will use information about the graph to write equations of particular linear functions.

Objective:

Given information about the graph of a linear function, write its particular equation.

EXAMPLE 1

Find the particular equation of the linear function with slope $-\frac{3}{2}$, containing the point $(7, -5)$.

Solution:

Since a point and the slope are given, the easiest form to use is the point-slope form. You would write

$$y + 5 = -\frac{3}{2}(x - 7)$$

The "+" is used on the left since the left member must be 0 when y is -5. The "$-$" is used on the right for similar reasons. It is not necessary to transform to any other form unless you are asked to do so.

Find the particular equation of the linear function containing the points $(-4, 5)$ and $(6, 10)$.

Solution:
This new problem can be turned into an old problem by first using the slope formula to find the slope, m.

$$m = \frac{10 - 5}{6 - (-4)} = \frac{5}{10} = 0.5$$

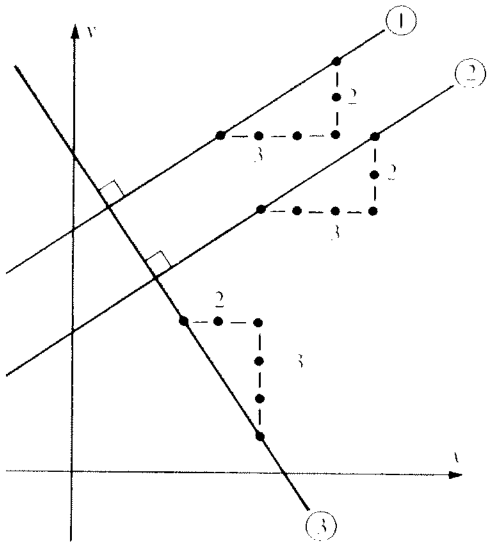
You can use either of the given points in the point-slope form.

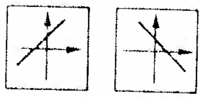
$$\underline{y - 5 = 0.5(x + 4)} \quad \text{or} \quad \underline{y - 10 = 0.5(x - 6)}$$

These two equations are equivalent, as you can see by transforming each to slope-intercept form or $Ax + By = C$ form.

$$y = 0.5x + 7 \quad \text{or} \quad x - 2y = -14$$

Two lines are *parallel* to each other if their slopes are equal. Figure 3-4a illustrates this fact. If the lines are perpendicular to each other, the slope of one is the *opposite* of the *reciprocal* of the other. For instance, the slope of Line (2) in Figure 3-4a is $\frac{2}{3}$. The slope of Line (3), perpendicular to Line (2), is $-\frac{3}{2}$. These facts can be used to find particular equations.





PROPERTY

PARALLEL AND PERPENDICULAR LINES
If the equation of a line is $y = mx + b$, then:

A **parallel** line also has slope m.

A **perpendicular** line has slope $\frac{-1}{m}$.

EXAMPLE 3

Find the particular equation of the linear function containing $(-2, 7)$ if its graph is perpendicular to the graph of $3x + 4y = 72$.

Solution:
Transforming $3x + 4y = 72$ to $y = mx + b$ gives

$$y = -\frac{3}{4}x + 18$$

The slope of the given line is $-\frac{3}{4}$. So the slope of the desired line must be $\frac{4}{3}$, the opposite of the reciprocal of $-\frac{3}{4}$. The particular equation is thus

$$y - 7 = \frac{4}{3}(x + 2)$$

EXAMPLE 4

Find the particular equation of the horizontal line through $(7, 8)$.

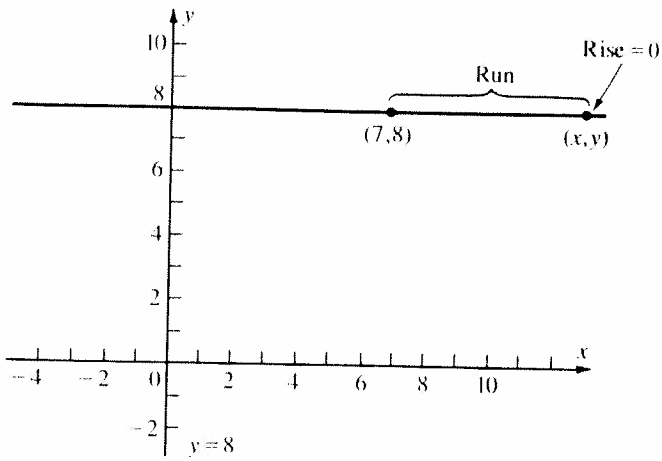


Figure 3-4b

Solution:
The easiest way to work this problem is to realize that horizontal lines have equations of the form y = constant. So you just write

$$\underline{\underline{y = 8.}}$$

The graph is shown in Figure 3-4b.

The problem can also be worked by realizing that the slope of a horizontal line is 0. Using the point-slope form gives

$$y - 8 = 0(x - 7),$$

which can be transformed to $y = 8$. ∎

EXAMPLE 5

Write the particular equation of the vertical line through (7, 8).

Solution:
The only way you can answer this question is to be brilliant, and just write down

$$\underline{\underline{x = 7.}}$$

Since the slope of a vertical line does not equal a real number, neither the point-slope nor the slope-intercept form can be used. The graph is shown in Figure 3-4c.

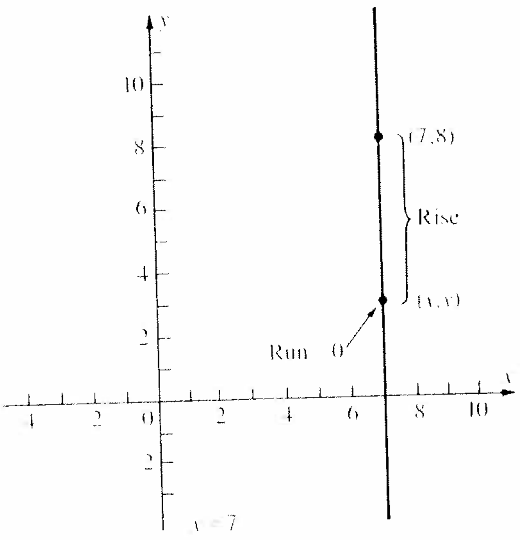
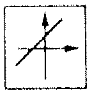
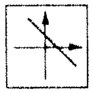


Figure 3-4c ∎

In the following exercise you will get practice writing equations if information about the graph is given. It is this technique that will let you use linear functions in the next section to represent situations from the real world.

EXERCISE 3-4

Do These Quickly

The following problems are intended to refresh your skills. You should be able to do all 10 in less than 5 minutes.

Q1. Write the general equation for slope-intercept form.

Q2. Find the x-intercept: $3x + 4y = 36$

Q3. Factor: $x^2 - x - 72$

Q4. Solve: $2x - 3 = 2(x + 4)$

Q5. 30 is 40% of what number?

Q6. Write the equation in the commutative axiom for addition.

Q7. Find the slope: $y = \frac{4}{7} + 3x$

Q8. Evaluate 5^3.

Q9. Divide $\frac{2}{3}$ by $\frac{3}{4}$.

Q10. Do the squaring: $(x - 3)^2$

Work these problems.

For Problems 1 through 26,

a. Write the particular equation of the line described.
b. Transform the equation (if necessary) to slope-intercept form.
c. Transform the equation to $Ax + By = C$ form, where A, B, and C are *integer* constants.

1. Has y-intercept of 21 and slope of -5.
2. Has y-intercept of -13 and slope of 7.
3. Contains (3, 7) and has a slope of 11.
4. Contains (4, 9) and has a slope of 5.73.
5. Contains (4, -5) and has a slope of -6.

6. Contains $(-3, 7)$ and has a slope of $-\frac{8}{5}$.
7. Contains $(1, 7)$ and $(3, 10)$.
8. Contains $(5, 2)$ and $(8, 11)$.
9. Contains $(2, -4)$ and $(-5, -10)$.
10. Contains $(-1, 4)$ and $(-5, -4)$.
11. Contains $(5, 8)$ and is parallel to the graph of $y = 7x - 6$.
12. Contains $(7, 2)$ and is parallel to the graph of $y = -4x + 3$.
13. Contains $(-4, 6)$ and is perpendicular to the graph of $y = 0.4x + 7$.
14. Contains $(3, -5)$ and is perpendicular to the graph of $y = -8x + 6$.
15. Contains $(5, 8)$ and is parallel to the graph of $2x + 3y = 9$.
16. Contains $(7, 2)$ and is parallel to the graph of $5x - 3y = 6$.
17. Contains $(4, 1)$ and is perpendicular to the graph of $5x - 7y = 44$.
18. Contains $(0, 6)$ and is perpendicular to the graph of $3x + 4y = 120$.
19. Has x-intercept of 5 and slope of $-\frac{2}{3}$.
20. Has x-intercept of 7 and y-intercept of 5.
21. Contains the origin, and has slope of 0.315.
22. Contains the origin, and has slope of 2.
23. Is horizontal, and contains $(-8, 9)$.
24. Is horizontal, and contains $(11, -13)$.
25. Is vertical, and contains $(-8, 9)$.
26. Is vertical, and contains $(11, -13)$.

For Problems 27 through 30, tell whether or not there is a linear function that contains *all* the points listed. If there is, find its particular equation.

27. $(6, 2), (5, 3), (1, 7)$
28. $(-3, 16), (1, 10), (9, -3)$
29. $(1, 4), (3, 7), (5, 10), (7, 13)$
30. $(4, 9), (20, 23), (13, 17), (29, 31)$
31. ***Intercept Form Problem*** Another form of the linear function equation is

$$\frac{x}{a} + \frac{y}{b} = 1,$$