cx 11/8, 1-29-13

MATH 115 PRACTICE PROBLEMS SOLUTION SET Profs. Boorstein and Zaleskas 12/2010

The final examination will include mainly problems which are more or less similar in type and content to the ones herein. Problems of other sorts may appear as well. The actual final examination may include other problems which are suitable for Mathematics 115 but are not exactly like any of the problems herein.

You must show your work for each problem in order to receive credit for it. All examples must be done algebraically. No credit will be given for answers obtained by trial and error.

1. (a)Simplify. Express all answers with positive exponents and exactly evaluate all numbers. Assume all letters represent positive quantities.

i) $\left(\frac{9a^{-3}b^{2}c^{-4}}{54a^{6}b^{-4}c^{2}}\right)^{-2} = \left(\frac{54a^{6}b^{-4}c^{2}}{9a^{-3}b^{2}c^{-4}}\right)^{2} = \left(\frac{6a^{9}c^{6}}{b^{6}}\right)^{2} = \frac{36a^{18}c^{12}}{b^{12}}$

ii) $\left(3a^{2/5}b^{-4}\right)^{-1/2} = \frac{1}{(3a^{2/5}b^{-4})^{1/2}} = \frac{1}{3^{\frac{1}{2}}a^{\frac{1}{5}}b^{-2}} = \frac{b^{2}}{3^{\frac{1}{2}}a^{\frac{1}{5}}}$

or $\frac{b^2}{\sqrt{3}\sqrt[5]{a}}$

iii) $\left(\frac{8}{27}\right)^{\frac{2}{3}} \cdot 2^{-4} = \left[\left(\frac{2}{3}\right)^{3}\right]^{\frac{2}{3}} \cdot 2^{-4} = \left(\frac{2}{3}\right)^{2} \cdot 2^{-4} = \frac{2^{2}}{3^{2}} \cdot 2^{-4} = \frac{2^{-2}}{3^{2}} = \frac{1}{2^{2}3^{2}}$

$= \frac{1}{6^{2}} = \frac{1}{36}$

(b) i) Fully simplify the radical expression

$$\sqrt[3]{16x^7y^5}$$

$$= \sqrt[3]{2^4 x^6 x y^3 y^2} = \sqrt[3]{2^3 \cdot 2 \cdot x^6 \cdot x \cdot y^3 \cdot y^2} = \sqrt[3]{2^3 x^6 y^3}\ \sqrt[3]{2xy^2}$$

$$= 2x^2y\sqrt[3]{2xy^2}$$

ii) Express your answer to part a (ii) in fully simplified radical form.

$$\frac{b^2}{3^{\frac{1}{2}}a^{\frac{1}{5}}} = \frac{b^2}{\sqrt{3}\ \sqrt[5]{a}} = \frac{b^2}{\sqrt{3}\ \sqrt[5]{a}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt[5]{a^4}}{\sqrt[5]{a^4}}$$

$$= \frac{b^2\sqrt{3}\ \sqrt[5]{a^4}}{3a}$$

2. Compute and write the answer in scientific notation.

$$\frac{(6.4 \times 10^{120}) \times (2.1 \times 10^{-209})}{(7.0 \times 10^{-200}) \times (8.0 \times 10^{115})}$$

$$\frac{(\cancel{6.4}^{\,0.8} \times 10^{120}) \times (\cancel{2.1}^{\,0.3} \times 10^{-209})}{(\cancel{7.0} \times 10^{-200}) \times (\cancel{8.0} \times 10^{115})}$$

$$= \frac{0.8 \times 0.3 \times 10^{120-209}}{10^{-200+115}} = \frac{0.24 \times 10^{-89}}{10^{-85}} = 0.24 \times 10^{-89+85}$$

$$= 0.24 \times 10^{-4} = 2.4 \times 10^{-5}$$

3. Factor completely.

(a) $4t^3 + 108$ $= 4(t^3 + 27) = 4(t^3 + 3^3) = 4(t+3)(t^2 - 3t + 9)$

(b) $x^4 - 12x^2 + 27$ $= (x^2 - 9)(x^2 - 3) = (x+3)(x-3)(x^2 - 3)$

(c) $7x^2 - 9xy + 2y^2$ $= (7x - 2y)(x - y)$

(d) $x^3 - 3x^2 - 4x + 12$ $= x^2(x-3) - 4(x-3) = (x^2 - 4)(x-3)$
$= (x-2)(x+2)(x-3)$

4. (a) Perform the indicated operations and simplify: $(4x+7y)^2 - (2x+7y)(8x+7y)$

$$= 16x^2 + 56xy + 49y^2 - (16x^2 + 70xy + 49y^2)$$

$$\begin{array}{r} 56xy \\ \underline{14xy} \\ 70xy \end{array}$$

$$= 56xy - 70xy = -14xy$$

(b) rationalize the denominator and simplify: $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}$

$$\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} \cdot \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})}{5-3}$$

$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{2}$$

5. Solve for x and check.

$$\frac{x+3}{x-2} + \frac{x+1}{x+2} = \frac{8+6x}{x^2-4}$$

$LCD = (x+2)(x-2)$, M_{LCD}

$$(x+2)\cancel{(x-2)} \cdot \frac{x+3}{\cancel{(x-2)}} + \cancel{(x+2)}(x-2) \cdot \frac{x+1}{\cancel{x+2}} = \frac{8+6x}{x^2-4} \cdot (x+2)(x-2)$$

$$(x+2)(x+3) + (x-2)(x+1) = 8+6x$$

$$x^2+5x+6+x^2-x-2 = 8+6x$$

$$2x^2-2x-4=0 \qquad D_2$$

$$x^2-x-2=0$$

$$(x-2)(x+1)=0$$

$$x = 2, -1$$

$x=2$ extraneous root, not allowed substitution in original equation. Only solution is -1:

$$\{-1\}$$

Check: $\frac{-1+3}{-1-2} + \frac{-1+1}{-1+2} \stackrel{?}{=} \frac{8-6}{1-4}$

$$-\frac{2}{3} + 0 = \frac{2}{-3} \checkmark$$

6. Solve by quadratic formula.

$3(x^2 + 1) = 5(1 - x)$

$\rightarrow 3x^2 + 3 = 5 - 5x$

$3x^2 + 5x - 2 = 0$

$x = \frac{-5 \pm \sqrt{25 - 4(3)(-2)}}{2(3)} = \frac{-5 \pm \sqrt{25+24}}{6}$

$= \frac{-5 \pm \sqrt{49}}{6} = \frac{-5 \pm 7}{6} = -\frac{12}{6}, \frac{2}{6}$ or $-2, \frac{1}{3}$

7. Solve for x.

$\sqrt{2x - 2} + \sqrt{x - 2} = 3$

$\sqrt{2x-2} = 3 - \sqrt{x-2}$

$2x - 2 = 9 - 6\sqrt{x-2} + x - 2$

$x - 9 = -6\sqrt{x-2}$

$x^2 - 18x + 81 = +36x - 72$

$x^2 - 54x + 153 = 0$

$(x-3)(x-51) = 0$ $\quad x = 3$ or $x = 51$

$153 = 3 \cdot 51 = 3 \cdot 3 \cdot 17$

reject $x = 51$ extraneous

$\sqrt{102-2} \stackrel{?}{=} 3 - \sqrt{49}$

$10 \stackrel{?}{=} 3 - 7$

$10 \neq -4$

check $x = 3$

$\sqrt{4} \stackrel{?}{=} 3 - \sqrt{1}$

$2 = 3 - 1$

$2 = 2$ ✓

8. Find the centre and radius. $x^2 + y^2 - 16x + 14y + 17 = 0.$

$x^2 + y^2 - 16x + 14y + 17 = 0$

$(x^2 - 16x \quad) + (y^2 + 14y \quad) = -17$

$(x^2 - 16x + 64) + (y^2 + 14y + 49) = 64 + 49 - 17$

$(x-8)^2 + (y+7)^2 = 96$

$C(8, -7)$, $r = \sqrt{96} = \sqrt{16 \cdot 6} = 4\sqrt{6}$

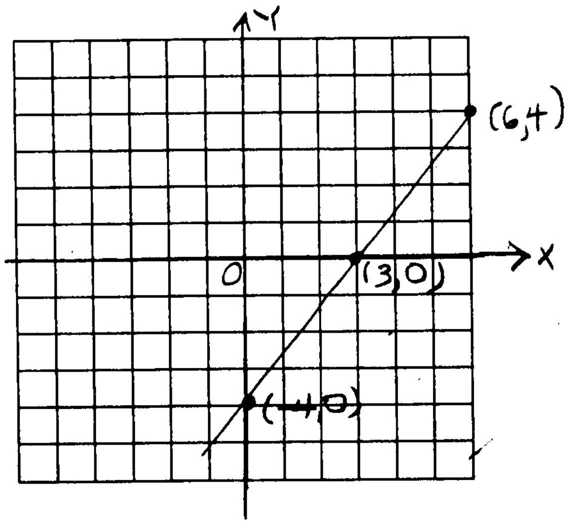
$$\begin{array}{r} 64 \\ +49 \\ \hline 113 \\ -17 \\ \hline 96 \end{array}$$

9. **(a) Find the slope and both intercepts:**

$\frac{x}{3} - \frac{y}{4} = 1$ $-\frac{y}{4} = -\frac{x}{3} + 1 \Rightarrow y = \frac{4}{3}x - 4$ $m = \frac{4}{3}$

(0, -4) y-intercept
(3, 0) x-intercept

(b) Graph the equation and label with their coordinates both intercepts, and the point where y = 4.



$4 = \frac{4}{3}x - 4$

$\frac{4}{3}x = 8$

$x = \frac{3}{4} \times 8 = 6$

$\rightarrow$ pt is (6, 4)

10. Find an equation of the line passing through (3,4) and perpendicular to $2x - 3y + 4 = 0$.
Write your answer in (a) point-slope form (b) slope-intercept form and (c) general form.

$-3y = -2x + 4 \Rightarrow y = \frac{2}{3}x - \frac{4}{3} \Rightarrow m = \frac{2}{3} \Rightarrow m_{\perp} = -\frac{3}{2}$

$\perp$ line is $y - 4 = -\frac{3}{2}(x - 3)$ point-slope form

$y = -\frac{3}{2}x + \frac{9}{2} + 4$

$y = -\frac{3}{2}x + \frac{17}{2}$ slope-intercept form

$2y = -3x + 17$

$3x + 2y = 17$ general form

11. Let $f(x) = {}^{1}/_{(x-1)}$, $g(x) = \sqrt{x}$. Find each of the following and rationalize any denominators:

(a) $(f+g)(x)$ $\quad (f+g)(x) = \dfrac{1}{x-1} + \sqrt{x}$

(b) $(f-g)(x)$ $\quad (f-g)(x) = \dfrac{1}{x-1} - \sqrt{x}$

(c) $(f \cdot g)(x)$ $\quad (f \cdot g)(x) = \dfrac{\sqrt{x}}{x-1}$

(d) $(f/g)(x)$ $\quad (f/g)(x) = \dfrac{\frac{1}{x-1}}{\sqrt{x}} = \dfrac{1}{\sqrt{x}(x-1)}$ = if rationalized, $\dfrac{\sqrt{x}}{x(x-1)}$

(e) $(g/f)(x)$ $\quad (g/f)(x) = \dfrac{\sqrt{x}}{\left(\frac{1}{x-1}\right)} = (x-1)\sqrt{x}$

12. Let $f(x) = \frac{1}{x-2}$, $g(x) = \sqrt{x}$. Find each of the following and rationalize any denominators:

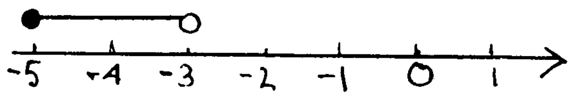
(a) $(f \circ g)(x)$ $\quad = f(g(x)) = \dfrac{1}{g(x)-2} = \dfrac{1}{\sqrt{x}-2} = \dfrac{1}{\sqrt{x}-2} \cdot \dfrac{\sqrt{x}+2}{\sqrt{x}+2} = \left(\dfrac{\sqrt{x}+2}{x-4}\right)$

(b) $(f \circ g)(9)$ $\quad = \dfrac{1}{\sqrt{9}-2} = \dfrac{1}{3-2} = 1$

(c) $(g \circ f)(x)$, $\quad = \sqrt{f(x)} = \sqrt{\dfrac{1}{x-2}} = \dfrac{1}{\sqrt{x-2}} = \dfrac{1}{\sqrt{x-2}} \cdot \dfrac{\sqrt{x-2}}{\sqrt{x-2}} = \dfrac{\sqrt{x-2}}{x-2}$

(d) $(g \circ f)(6)$. $\quad = \dfrac{1}{\sqrt{6-2}} = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2}$

13. (a) Write graphical notation for the interval $[-5,-3)$.



(b) Write interval notation for the set $\{x|x \leq 2\}$.

$(-\infty, 2]$

(c) Write set notation for the interval $(-5,\ 5)$

$\{x \mid -5<x<5\}$

14. (a) Use completing the square to find the vertex. (*No credit if any other method used.*)

$y = \frac{1}{2}x^2 + 2x + 1. = (\frac{1}{2}x^2 + 2x \quad) + 1 = \frac{1}{2}(x^2 + 4x \quad) + 1$

$= \frac{1}{2}(x^2+4x+4-4)+1 = \frac{1}{2}(x+2)^2 + \frac{1}{2}(-4)+1$

$= \frac{1}{2}(x+2)^2 - 1$ $\quad$ $V(-2,-1)$

(b) state the axis of symmetry $X = -2$

(c) find the coordinates of all intercepts, if any exist

$\frac{1}{2}(x+2)^2 - 1 = 0 \Rightarrow (x+2)^2 = 2 \Rightarrow x+2 = \pm\sqrt{2} \Rightarrow x = -2 \pm \sqrt{2}$

or $(-2 \pm \sqrt{2}, 0)$ are x-intercepts

$f(0) = 1$ y-intercept

i.e. $(0, 1)$

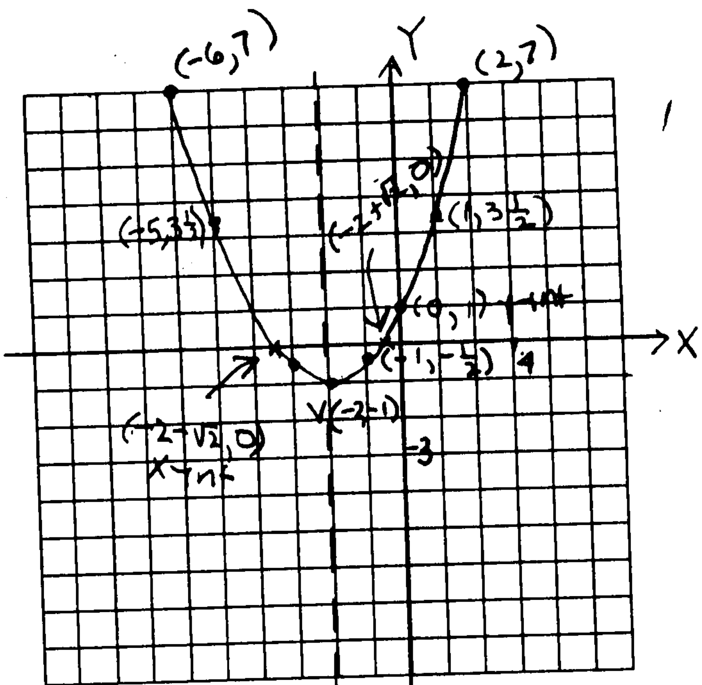
(d) graph the equation. Label with both their coordinates the vertex, any intercepts, and two other points and label the axis of symmetry.

$f(2) = \frac{1}{2} \cdot 2^2 + 2 \cdot 2 + 1 = 7$

by symmetry, $f(-6) = 7$

$f(-1) = \frac{1}{2} - 2 + 1 = -\frac{1}{2} = f(-3)$ by symm.

$f(1) = \frac{1}{2} + 2 + 1 = 3\frac{1}{2} = f(-5)$



← axis of symmetry
$x = -2$

15. Write the domain in interval form:

(a) $f(x) = \sqrt{42 - 7x}$ $\quad 42 - 7x \geq 0 \Rightarrow x \leq 6$ i.e. $(-\infty, 6]$

(b) $g(x) = \frac{1}{x^2 - 4x - 21}$ $\quad \{x \mid x^2 - 4x - 21 \neq 0\}$ i.e. $(x-7)(x+3) \neq 0$

or $\{x \mid x \neq -3, 7\}$; $(-\infty, -3) \cup (-3, 7) \cup (7, \infty)$

set notation   interval notation

(c) $h(x) = \frac{\sqrt{x+7}}{x^2 - 4x - 21}$ $\quad \frac{\sqrt{x+7}}{x^2 - 4x - 21}$

set notation

$\{x \mid x \geq -7$ and $x \neq 7, -3\}$

Interval notation:

$[-7, -3) \cup (-3, 7) \cup (7, \infty)$

16. Solve the system of equations. (*Must be done algebraically. Trial-and-error not acceptable.*)

$$\left.\begin{array}{l} 300x + 400y = 2{,}000 \\ 4x + 5y = 26 \end{array}\right\} \Rightarrow \begin{array}{l} 3x + 4y = 20 \xrightarrow{M_{-4}} \\ 4x + 5y = 26 \xrightarrow[M_3]{} \end{array}$$

$$\left.\begin{array}{l} -12x - 16y = -80 \\ 12x + 15y = 78 \end{array}\right\}$$ add 2 equations: $-y = -2$

or $y = 2 \Rightarrow 4x + 5(2) = 26$

$4x = 16$

$x = 4$

Solution $(4, 2)$

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WORD PROBLEMS

17. Plane B travels at 500 mph, while plane D travels at 302 mph. Plane D leaves an airport first. One hour later, plane B departs the same airport taking the same route. How long will it take B to overtake plane D? (*Express answer first exactly as a rational number, then as its decimal equivalent to the nearest 0.01 hours.*)

Let t = time for plane D in hours.

	R	T	D
plane B	500	$t-1$	$500(t-1)$
plane D	302	t	$302t$

They travel the same distance $\Rightarrow 500(t-1) = 302t$

$500t - 500 = 302t$

$198t = 500$

$t = \frac{500}{198} = \frac{250}{99}$ hr

$\Rightarrow t - 1 = \frac{250}{99} - 1 = \frac{151}{99}$ hr ≈ 1.53 hr

$=$ plane B's time

18. A snack food manufacturer wishes to mix sesame sticks worth $2.50 /lb. with Brazil nuts worth $7.50/lb in order to make 20 lb. of a mixture worth $4.50/lb..How much of each component should be used?

Let S = number of pounds of sesame sticks
and b = number of pounds of Brazil nuts

$$\begin{cases} S + b = 20 \quad ① \text{(conservation of mass)} \\ 2.50S + 7.50b = 20 \times 4.50 \quad ② \text{(conservation of money)} \end{cases}$$

from ① $\quad S = 20 - b$; substitute into ②.

$$2.50(20-b) + 7.50b = 90$$

$$50 - 2.50b + 7.50b = 90$$

$$5.00b = 40$$

$$b = \frac{40}{5} = 8\# \text{ of Brazil nuts}$$

$$S = 20 - b = 20 - 8 = 12\# \text{ of sesame sticks}$$

19. Al can paint a room in 1.5 hr.,Betty can paint the same room in 2.5 hours, and Vic can paint the same room in 3.5 hr..How long would it take to paint the room if all three worked together?

Al's rate is $\frac{1}{1.5}$ rooms/hr, Betty's is $\frac{1}{2.5}$ rooms/hr and Vic's is $\frac{1}{3.5}$ rooms/hr. Let t = time in hr for all 3 together to paint room.

$$\frac{1}{1.5} + \frac{1}{2.5} + \frac{1}{3.5} = \frac{1}{t}$$

$$\frac{1}{3/2} + \frac{1}{5/2} + \frac{1}{7/2} = \frac{1}{t}$$

$$\frac{2}{3} + \frac{2}{5} + \frac{2}{7} = \frac{1}{t} \qquad \text{LCD} = 3 \times 5 \times 7t = 105t$$

$$105t\left(\frac{2}{3}\right) + 105t\left(\frac{2}{5}\right) + 105t\left(\frac{2}{7}\right) = 105t\left(\frac{1}{t}\right)$$

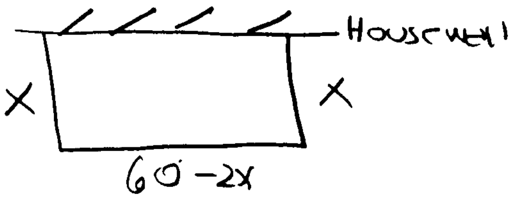
$$70t + 42t + 30t = 105$$

$$142t = 105$$

$$t = \frac{105}{142} \text{ hr} \approx 0.7394 \text{ hr}$$

70
42
30
142

20. A bricklayer has sufficient cinder blocks to enclose a rectangular patio with 60 feet of block wall. If the wall of the adjacent house is used to form one side of the rectangle, find (a) the maximum area that can be enclosed and (b) the dimensions of the patio needed in order to achieve this area. (*carefully identify the two dimensions in relation to the house wall.*)



Let x = dimension of rectangle in feet, $\perp$ to house wall

Then $60-2x$ is dimension parallel to house wall

Enclosed area = $A = x(60-2x)$, to be maximized

$$60x - 2x^2 = -2(x^2-30x) = -2(x^2-30x+225-225)$$

$$= -2(x-15)^2 + (-2)(-225)$$

$$= -2(x-15)^2 + 450.$$ Maximum area is $450\ \text{ft}^2$ (a)

and it occurs (b) when $x = 15'$ = dimension $\perp$ house,

$60-30 = 30'$ = dimension parallel to house

21. A vehicle travels 120 mi at a constant speed. If the speed had been 10 mi/hr faster, the travel time would have been 2 hours less. Find the speed of the vehicle.

Let v = speed of vehicle in mph

	R	T	D(mi)
original trip	v	$\frac{120}{v}$	120
faster trip	$v+10$	$\frac{120}{v+10}$	120

$$\frac{120}{v+10} = \frac{120}{v} - 2$$, LCD $= v(v+10)$

$$120v = 120(v+10) - 2v(v+10)$$

$$120v = 120v + 1200 - 2v^2 - 20v$$

$$2v^2 + 20v - 1200 = 0$$

$$v^2 + 10v - 600 = 0$$

$$(v+30)(v-20) = 0$$

$v = 20$ mph = original speed of vehicle. $v = -30 < 0$ is extraneous root.