Function Composition and Function Inverses: Notes 6/27/2016

I. Function composition

Suppose f(x) and g(x) are functions.

Then the composite function $f^{o}g(x)$, called "the composition of f and g", is defined by:

$f^{o}g(x) = f(g(x))$, **whenever it is defined.**

Notice that the symbol in between f and g is a **raised circle** (not a solid dot).

Example 1. $f(x) = 2x+1$, and $g(x) = x^2$.

Since the "variable" "x" is just a placeholder, the formula "f(x) = 2x+1" could just as well be written "f(t) = 2t +1" or "f(w) = 2w +1"; similarly for $g(x) = x^2$ which could be written $g(p)=p^2$.

The issue here is that the "x" in f(x) and the "x" in g(x) *are not the same x* !

Evaluating, $f^{o}g(x) = f(g(x)) = f(x^2) = 2(x^2) + 1$.

However, $g^{o}f(x) = g(f(x)) = g(2x+1) = (2x+1)^2 = 4x2+4x+1$.

NOTICE: $g^{o}f$ and $f^{o}g$ are **not the same function**! Sometimes they could be, but usually they are not. That is to say, function composition is **not commutative**!

But function composition is **associative**: that is, $(f^{o}g)^{o}h$ is always the same function as $f^{o}(g^{o}h)$.

Sometimes, finding the domain of $f^{o}g$ can be a bit tricky. Remember that in order for x=a to be in the domain of $f^{o}g$, f(a) must be in the domain of the function g.

II. Function Inverse

The functions f and g are called "inverse functions" iff:

The function composition $(f^{o}g)(x) = x$, and also the function composition $(g^{o}f)(x) = x$.

Notation: When f and g are inverses of each other, we write $f^{-1}(x) = g(x)$, and also $g^{-1}(x) = f(x)$.

Example 2. Let f(x) = (2x+6)/5. Let's examine what happens to "x" according to the rules for order of operations:

Start with x	Say, x = 7	7
Multiply by 2	2*7	14
Add 6	2*7 + 6	20
Divide by 5	(2*7 + 6)/5	4

Now, suppose we want to UNDO this process. Then, we need to perform, step-by-step, the INVERSE operations in the OPPOSITE order. Before we do this, here is a table of operations and their inverses:

OPERATION	INVERSE OPERATION
Put on a shoe	Take off a shoe
Add 5	Subtract 5
Subtract 17	Add 17
Multiply by 9	Divide by 9
Divide by 3	Multiply by 3
Square the number	Take the square root of the number ***[CAUTION: THIS IS NOT ACTUALLY AN INVERSE, since $(- 5)^2 = 25$, but √(25) = +5, so you don't get back to "-5".***]
Take the reciprocal of [that is, x→ (1/x)]	Take the reciprocal of [that is, x→ (1/x)]
Take the additive inverse of the number	Take the additive inverse of the number
Take $\log_b( )$ of the number	Take $b^{( )}$ of the number

Now, finding the INVERSE function to f(x) = (2x+6)/5 by the **<u>VERBAL STRING METHOD</u>**:

Direction for finding f(x)	f(x) in steps		Direction for finding $f^{-1}(x)$	Inverse operations of the steps	Inverse of f(x) in steps
↓	Start with x	x	↑		
	Multiply by 2	2x		Divide by 2	(5x-6)/2
	Add 6	2x+6		Subtract 6	(5x-6)
	Divide by 5	(2x+6) / 5		Multiply by 5	5x
					x
					START HERE

By the above chart, the inverse function f-1(x) = (5x-6)/2 .

III. **<u>Algebraic method</u>** of finding function inverse

Suppose now that f(x) = [3x+5] / [2x-4]. We can't use the Verbal String method, since there are TWO occurrences of "x" in the formula for f(x). Instead , use this algebraic method:

START: Given	f(x) = [3x+5] / [2x-4].
Step 1. Replace "f(x)" by "y".	y = [3x+5] / [2x-4].
Step 2. Solve for x in terms of y.	y [2x-4]. = [3x+5] . 2xy -4y = 3x + 5 2xy – 3x = 4y +5 x[2y – 3] = 4y +5 x = [4y +5] / [2y -3]
Step 3: Interchange y and x.	y = [4x +5] / [2x -3]
Step 4: Replace "y" by " $f^{-1}(x)$"	$f^{-1}(x)$ = [4x +5] / [2x -3]
END OF ALGORITHM	