Factoring a polynomial over the integers, in one variable:

Perform these steps repeatedly in order until you are done with each step.

1. **Simplify** by combining like terms; then write in **descending order.**
2. Factor out the **greatest common monomial factor**.
3. **Difference of two squares**.
4. **Difference of two like odd powers**: $a^5-b^5 = (a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$
5. **Sum of two like odd powers**: $a^5+b^5 = (a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$
6. **Trinomial: ax^2+bx+c = (maybe) $(dx+e)(fx+g)$**
    a. If needed, factor out a (-1) so that the term of highest degree is positive.
    b. If **c is positive** then e and g both have the **same sign**.
        i. Then, if b is positive, e and g are both positive.
        ii. Or, if b is negative, e and g are both negative.
    c. If c is negative, then e and g have opposite signs.
    d. Write all possible factor pairs of a; these are possible values of d and f.
    e. Write all possible factor pairs of c; these are possible values of e and g.
    f. Try all possible combinations of $(dx+e)(fx+g)$. If they fail, there is no factorization over the integers.
    g. Use Dr. L's divisibility trick. If you already did step (2) completely, then it is not possible for d and e to have a common factor; and it is not possible for f and g to have a common factor. This is true because any common factor of (say) d and e would also be a factor of every term of the original trinomial. This way, you can sometimes reject candidate factors.
    h. To shortcut your work, use the discriminant $\Delta = b^2 -4ac$ of the trinomial. The trinomial factors over the integers iff Δ is a (positive) perfect square.
7. Further work on trinomials: You can solve the equation $\mathbf{ax^2+bx+c =0}$ by using the **quadratic formula.** If r1 and r2 are the roots of that equation, then (x-r1) and (y-r2) are factors [but you might have to multiply these factors by some integers to get the correct answer!
8. #7 is a special case of the Factor Theorem: if P(x) is any polynomial in x, then:
    a. (x-r) is a factor of P(x) iff P(r) = 0.
    b. A corollary of the Factor Theorem is the Rational Root theorem.
9. Special Products and Grouping. Examples of these are omitted here.

Factoring a polynomial in two variables:

10. **$ax^2+bxy +cy^2$ may be factored as follows:**
    a. **Set y=1. Then factor ax^2+bx+c (if possible) as** $(dx+e)(fx+g)$.
    b. **Then the factors of $ax^2+bxy +cy^2$ factors as:** $(dx+ey)(fx+gy)$.
    c. **Example:** $4x^2+4xy+y^2$. Then $4x^2+4x+1 = (2x+1)(2x+1)$. So, $4x^2+4xy+y^2 = (2x+y)(2x+y)$.

Factoring a polynomial (of higher degree) over $\mathbb{R}$ or the $\mathbb{C}$:

11. Sketch the polynomial, picking points wisely. Find the critical points if possible. Use the fundamental theorem of algebra. And, if the polynomial is positive at $x=x_1$ and negative at $x=x_2$, then it must take the value zero somewhere between x_1 and x_2. As you find factors, use polynomial long division to reduce the degree of the polynomial.

How to factor (almost) anything. Name________________________ Section ______________

Directions: Factor completely over the rational numbers. Check by multiplying! If the expression cannot be factored, write "prime". Do these problems in your HW book. Copy answers here.

	Problem	Work	Answer
1	x^2-9	Difference of 2 squares	(x-3)(x+3)
2	$4x^2$-25	Difference of 2 squares	(2x-5)(2x+5)
3	49-$9x^2$	Difference of 2 squares	(7-3x)(7+3x)
4	1-$169x^2$	Difference of 2 squares	(1-13x)(1+13x)
5	$3x^3$-27x	CMF, Difference of 2 squares; 3x(x^2-9)	(3x)(x+3)(x-3)
6	$5x^4$ -625x	CMF; 5x(x^3-125); Difference of 2 cubes	5x(x-5)(x^2+5x+25)
7	$(x+7)^2$-16	Difference of 2 squares. ((x+7)-4)((x+7)+4)	(x+3)(x+11)
8	x^5-32x	CMF: x(x^4-32)	x(x^4-32)
9	x^2+12	Trinomial; Δ = b^2-4ac=$(0)^2$-4(1)(12)= -48	prime
10	x^2-11	Trinomial; Δ = b^2-4ac=$(0)^2$-4(1)(-11)= 44	prime
11	x^2-3x+6-2x	Simplify to x^2-5x+6; trinomial.	(x-2)(x-3)
12	x^2-9x+20	Trinomial.	(x-5)(x-4)
13	$6w^2$-7w-20	Trinomial. Δ = b^2-4ac=$(-7)^2$-4(6)(-20)= 49+480=529=23^2; Factors.	(3w+4)(2w-5)
14	$5wx^2$-110wx+605w	CMF: 5w(x^2-22x+121).2nd factor is a square.	5w$(x-11)^2$
15	x^3+y^3	Sum of 2 cubes.	(x+y)(x^2-xy+y^2)
16	x^3-8	Difference of 2 cubes x^3-2^3	(x-2)(x^2+2x+4)
17	$5x^6y^6$ -$625x^3y^3$	CMF: $5x^3y^3$(x^3y^3-125). Second factor is a difference of 2 cubes ($(xy)^3$-5^3).	$5x^3y^3$ (xy-5) (x^2y^2+5xy+25)
18	x^2-5x+6	Trinomial	(x-3)(x-2)
19	$6x^2$+11x-10	Trinomial. Δ = b^2-4ac=$(11)^2$-4(6)(-10)= 121+240=361 =19^2. Factors.	(2x+5)(3x-2)
20	x^2+x+3	Trinomial. Δ = b^2-4ac=$(1)^2$-4(1)(3)= -11.	Prime
21	x^3-$49x^2$-343x	CMF: x(x^2-49x-343). Δ = b^2-4ac=$(-49)^2$-4(1)(-343)=2401+1372=3773, not a square.	x(x^2-49x-343)
22	$8x^4$+64x	CMF: 8x(x^3+8). Sum of 2 cubes.	8x(x+2)(x^2-2x+4)
23	$8x^4$+81x	CMF: x($8x^3$+81). 2nd factor is prime.	x($8x^3$+81).
24	$(2x+1)^2$ -3(2x+1)+2 [Hint: let p = (2x+1)]	p^2-3p+2 factors as (p-1)(p-2). Substitute (2x+1) for p. ((2x+1)-1)((2x+1-2)	2x(2x-1)
25	$6x^2$-5x-6	Trinomial. Δ = b^2-4ac=$(-5)^2$-4(6)(-6) =25+144=169=13^2. Factors.	(3x+2)(2x-3)
26	$6x^2$-7x-6	Trinomial. Δ = b^2-4ac=$(-7)^2$-4(6)(-6) = 49+144=193=not a square. prime.	Prime.
27	$5x^2$-35x+65	CMF.Trinomial Δ = b^2-4ac=$(-7)^2$-4(1)(13) = -3	5(x^2-7x+13)
28	w^9-x^3	difference of 2 cubes = $(w^3)^3$-x^3	(w^3-x)(w^6+w^3x+x^2)
29	$16x^2$-8xy+y^2	Trinomial, perfect square	$(4x-y)^2$
30	x^6-y^6	Diff. of 2 sqs=(x^3-y^3)(x^3+y^3). Factors further.	(x-y) (x^2+xy+y^2) (x+y)(x^2-x+y^2)
31	x^4-$3x^3$-x^2+3x = P(x). Hint: (x-3)	CMF x. P(3)=0. (x-3) is a factor. Divide. P(x)= x[x-3][x^2-1] =	x(x-3)(x+1)(x-1)
32	x^4-$4x^3$-x^2+16x-12 Hint: factor theorem	Factor th. Try ±1,2,3,4,6,12.P(1)=0. P(2)=0. Divide: P(x)=(x-2)(x-1)(x^2-x-6).	(x-1)(x-2)(x-3)(x+2)