

Part I. transforming a quadratic equation from polynomial form to standard (vertex) form.

	Polynomial form	Work	Vertex form																				
0	$y=ax^2+bx+c$	<div><div>[example] We want to get the equation into the form $(y-k) = a(x-h)^2$ Note that the vertex is (h,k) in the above equation.</div><table><tr><td>Original equation</td><td>$y = ax^2+bx+c$</td></tr><tr><td>Move c to the left</td><td>$y-c = ax^2+bx$</td></tr><tr><td>Factor out a from LHS</td><td>$y-c = a [x^2+(b/a)x \quad]$</td></tr><tr><td>Complete the square inside the big [], adding and subtracting</td><td>$y-c = a [x^2+(b/a)x + (b/2a)^2 - (b/2a)^2 \quad]$</td></tr><tr><td>Write the 1st 3 terms in [] as a square</td><td>$y-c = a [(x+(b/2a))^2 - (b/2a)^2 \quad]$</td></tr><tr><td>Distribute “a” to both terms on the right</td><td>$y-c = a [(x+(b/2a))^2] - a(b/2a)^2$</td></tr><tr><td>Move the constant term to the left</td><td>$y-c + a(b/2a)^2 = a [(x+(b/2a))^2 \quad]$</td></tr><tr><td>Simplify the third term on the right</td><td>$y-c + b^2/4a = a [(x+(b/2a))^2 \quad]$</td></tr><tr><td>Combine terms on the left and use “-” signs after both variables.</td><td>$y-c(4a/4a) + b^2/4a = a [(x+(b/2a))^2 \quad]$ $y - (b^2-4ac)/4a = a [(x+(-b/2a))^2 \quad]$</td></tr><tr><td></td><td>Vertex = $(-b/2a, -\Delta/4a \quad)$</td></tr></table></div>	Original equation	$y = ax^2+bx+c$	Move c to the left	$y-c = ax^2+bx$	Factor out a from LHS	$y-c = a [x^2+(b/a)x \quad]$	Complete the square inside the big [], adding and subtracting	$y-c = a [x^2+(b/a)x + (b/2a)^2 - (b/2a)^2 \quad]$	Write the 1 st 3 terms in [] as a square	$y-c = a [(x+(b/2a))^2 - (b/2a)^2 \quad]$	Distribute “a” to both terms on the right	$y-c = a [(x+(b/2a))^2] - a(b/2a)^2$	Move the constant term to the left	$y-c + a(b/2a)^2 = a [(x+(b/2a))^2 \quad]$	Simplify the third term on the right	$y-c + b^2/4a = a [(x+(b/2a))^2 \quad]$	Combine terms on the left and use “-” signs after both variables.	$y-c(4a/4a) + b^2/4a = a [(x+(b/2a))^2 \quad]$ $y - (b^2-4ac)/4a = a [(x+(-b/2a))^2 \quad]$		Vertex = $(-b/2a, -\Delta/4a \quad)$	
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2	$y = -2x^2 -3x -1$																						
3	$y = 5x^2+x+2$																						