There are eleven axioms that apply to adding and multiplying real numbers. These are called the *Field Axioms*, and are listed in the following table. If you already feel familiar with these axioms, you may go right to the problems in Exercise 1-2. If not, then read on!

THE FIELD AXIOMS

CLOSURE

{real numbers} is *closed* under addition and under multiplication. That is, if x and y are real numbers, then

$x + y$ is a *unique, real* number;
xy is a *unique, real* number.

COMMUTATIVITY

Addition and multiplication of real numbers are *commutative* operations. That is, if x and y are real numbers, then

$x + y$ and $y + x$ are *equal* to each other,
xy and yx are equal to each other.

ASSOCIATIVITY

Addition and multiplication of real numbers are *associative* operations. That is, if x, y, and z are real numbers, then

$(x + y) + z$ and $x + (y + z)$ are *equal* to each other.
$(xy)z$ and $x(yz)$ are *equal* to each other.

DISTRIBUTIVITY

Multiplication *distributes* over addition. That is, if x, y, and z are real numbers, then

$x(y + z)$ and $xy + xz$ are *equal* to each other.

IDENTITY ELEMENTS

{real numbers} contains:

A *unique* identity element for *addition*, namely 0. (Because $x + 0 = x$ for any real number x.)

A *unique* identity element for *multiplication*, namely 1. (Because $x \cdot 1 = x$ for any real number x.)

INVERSES

{real numbers} contains:

A *unique additive* inverse for every real number x. (Meaning that every real number x has a real number $-x$ such that $x + (-x) = 0$.)

A *unique multiplicative* inverse for every real number x except zero.
(Meaning that every non-zero number x has a real number $\frac{1}{x}$ such that $x \cdot \frac{1}{x} = 1$.)

Notes:

1. Any set that obeys all eleven of these axioms is a *field*.
2. The eleven Field Axioms come in 5 pairs, one of each pair being for addition and the other for multiplication. The Distributive Axiom expresses a relationship between these two operations.
3. The properties $x + 0 = x$ and $x \cdot 1 = x$ are sometimes called the "Addition Property of 0" and the "Multiplication Property of 1," respectively, for obvious reasons.
4. The number $-x$ is called, "the *opposite* of x," "the *additive inverse* of x," or "negative x."
5. The number $\frac{1}{x}$ is called the "multiplicative inverse of x," or the

examples first, then checking to be sure you are right. Correct any which you left out or got wrong.

4. Explain why 0 has *no* multiplicative inverse.

5. The Closure Axiom states that you get a *unique* answer when you add two real numbers. What is meant by a "unique" answer?

6. You get the same answer when you add a column of numbers "up" as you do when you add it "down." What axiom(s) show that this is true?

7. Calvin Butterball and Phoebe Small use the distributive property as follows:

   Calvin: $3(x + 4)(x + 7) = (3x + 12)(x + 7)$.
   Phoebe: $3(x + 4)(x + 7) = (3x + 12)(3x + 21)$.

   Who is right? What mistake did the other one make?

8. Write an example which shows that:
   a. Subtraction is *not* a commutative operation.
   b. {negative numbers} is *not* closed under multiplication.
   c. {digits} is *not* closed under addition.
   d. {real numbers} is *not* closed under the $\sqrt{\ }$ operation (taking the square root).
   e. Exponentiation ("raising to powers") is *not* an associative operation. (Try 4^{2^3}.)

9. For each of the following, tell which of the Field Axioms was used, and whether it was an axiom for *addition* or for *multiplication*. Assume that x, y, and z stand for real numbers.
   a. $x + (y + z) = (x + y) + z$
   b. $x \cdot (y + z)$ is a real number
   c. $x \cdot (y + z) = x \cdot (z + y)$
   d. $x \cdot (y + z) = (y + z) \cdot x$
   e. $x \cdot (y + z) = xy + xz$
   f. $x \cdot (y + z) = x \cdot (y + z) + 0$
   g. $x \cdot (y + z) + (-[x \cdot (y + z)]) = 0$
   h. $x \cdot (y + z) = x \cdot (y + z) \cdot 1$
   i. $x \cdot (y + z) \cdot \dfrac{1}{x \cdot (y + z)} = 1$

10. Tell whether or not the following sets are *fields* under the operations + and ×. If the set is not a field, tell which one(s) of the Field Axioms do not apply.
    a. {rational numbers}
    b. {integers}
    c. {positive numbers}
    d. {non-negative numbers}

AGREEMENT

ORDER OF OPERATIONS

1. Do any operations inside parentheses *first*.
2. Do any exponentiating next.
3. Do multiplication and division in the order in which they occur, from left to right.
4. Do addition and subtraction last, in the order in which they occur, from left to right.

EXAMPLE 2

Carry out the following operations:

a. $3 + 4 \times 5$ — Multiply *first*.

$= 3 + 20$

$= \underline{\underline{23}}$ — Add *last*.

b. $3 + 4 \times 5 \div 2$

$= 3 + 20 \div 2$ — Multiply and divide from left to right.

$= 3 + 10$ — Divide *before* adding.

$= \underline{\underline{13}}$ — Add *last*.

c. $3 - 4 \times 5 \div 2 + 9$

$= 3 - 20 \div 2 + 9$ — Multiply and divide from left to right.

$= 3 - 10 + 9$ — Divide *before* + and −.

$= -7 + 9$ — Add and subtract from left to right.

$= \underline{\underline{2}}$ — Add and subtract last.

An expression might contain the *absolute value* operation. The symbol $|x|$ means the *distance* between the number x and the origin of the number line. For example, $|-3|$ and $|3|$ are both equal to 3, since both 3 and -3 are located 3 units from the origin (Figure 1-3).

Q7. Write the multiplicative identity element.

Q8. If $3x$ equals 42, what does x equal?

Q9. Multiply: (2.3)(4)

Q10. Divide and simplify: $(\frac{2}{3}) \div (\frac{6}{7})$

For Problems 1 through 10, carry out the indicated operations in the agreed-upon order.

1. $5 + 6 \times 7$
2. $3 + 8 \times 7$
3. $9 - 4 + 5$
4. $11 - 6 + 4$
5. $12 \div 3 \times 2$
6. $18 \div 9 \times 2$
7. $7 - 8 \div 2 + 4$
8. $24 - 12 \times 2 + 4$
9. $16 - 4 + 12 \div 6 \times 2$
10. $50 - 30 \times 2 + 8 \div 2$

For Problems 11 through 24, evaluate the given expression

(a) for $x = 2$
(b) for $x = -3$.

11. $4x - 1$
12. $3x - 5$
13. $|3x - 5|$
14. $|4x - 1|$
15. $5 - 7x - 8$
16. $8 - 5x - 2$
17. $|8 - 5x| - 2$
18. $|5 - 7x| - 8$
19. $x^2 - 4x + 6$
20. $x^2 + 6x - 9$
21. $4x^2 - 5x - 11$
22. $5x^2 - 7x + 1$
23. $5 - 2 \cdot x$
24. $3 + 4 \cdot x$

For Problems 25 through 40, simplify the given expression.

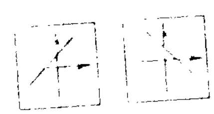
25. $6 - [5 - (3 - x)]$
26. $2x - [3x + (x - 2)]$
27. $7(x - 2(3 - x))$
28. $3(6x - 5(x - 1))$
29. $7 - 2[3 - 2(x + 4)]$
30. $8 + 4[5 - 6(x - 2)]$
31. $3x - [2x + (x - 5)]$
32. $4x - [3x - (2x - x)]$
33. $6 - 2[x - 3 - (x + 4) + 3(x - 2)]$
34. $7[2 - 3(x - 4) + 4(x - 6)]$
35. $6[x - \frac{1}{2}(x - 1)]$
36. $8[2x - \frac{1}{4}(6x + 5)]$

In the following exercise you will practice drawing linear graphs.

EXERCISE 3-2

For Problems 1 through 20, plot the graph neatly on graph paper. Use the slope and y-intercept, where possible.

1. $y = \frac{3}{5}x + 3$
2. $y = \frac{5}{2}x - 1$
3. $y = -\frac{3}{2}x - 4$
4. $y = -\frac{1}{4}x + 3$
5. $y = 2x - 5$
6. $y = 3x - 2$
7. $y = -3x + 1$
8. $y = -2x + 6$
9. $7x + 2y = 10$
10. $3x + 5y = 10$
11. $x - 4y = 12$
12. $2x - 5y = 15$
13. $y = 3x$
14. $y = -2x$
15. $y = 3$
16. $y = -5$



17. $x = -4$
18. $x = 2$
19. $y = 0$
20. $x = 0$
21. Relations such as in Problems 15 through 20 where