PRACTICE PROBLEMS FOR MATHEMATICS 129

1. If x units of a product can be sold at a price of p dollars per unit where $x + 2p = 200$, find the price(s), p, which will yield a revenue of \$4200.

2. It costs \$1000 to make 100 units of a certain product and \$1200 to make 150 units of the same product. Assume that the cost equation is linear.

   (a) Write the cost equation.

   (b) If the product can be sold at \$10 per unit, how many units must be made and sold to break even?

3. (a)Use completing the square to find the vertex of $y = 3 - 2x - x^2$.
   (b) Graph the curve. Label the vertex and at least two other points

4. Let $f(x) = \sqrt{x-3}$ and $g(x) = \dfrac{1}{x+1}$. Find:

   (a) domain of f

   (b) $f(g(x))$

   (c) $g(f(x))$

Do any four problems.

5. Solve for x:

   (a) $2^{x^2}2^{-2x} = 8$

   (b) $\log_3 3 + \log_3(x+1) - \log_3(2x-7) = 4$

   (c) $e^{2x} - e^{x} - 6 = 0$

   [Exact solutions, not calculator approximations.]

6. What monthly payment is required to pay off a loan of \$240,000 over 30 years at the nominal annual rate of 9% on the unpaid balance, with compounding at the end of each month?

7. Solve for x:

   (a) $x = \log_3 8$. (Get a number for your answer.)

   (b) $\log_2(x+1) = 4$.

8. How long would it take for \$5000 to grow to \$7000 if it is invested at a nominal annual rate of 6% compounded quarterly?

9. The demand equation for a certain product is $p = 200e^{-0.01x}$.

   (a) At what price p will 150 units be sold?

   (b) How many units (round off to the nearest unit) will be sold if the price per unit is $10?

10. Solve: $x^2 + 4x - 21 \geq 0$.

11. Given that $A = \begin{bmatrix} 5 & -8 \\ -4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -4 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 2 & -1 & 0 \end{bmatrix}$,

    find:

    (a) $3A$

    (b) $A + B$

    (c) AB

    (d) A^{-1}

    (e) C^{-1}

12. Solve using matrices and row-reduction. If there is no solution, indicate why. If there are an infinite number of solutions, give the general form of the solution.

$$\begin{aligned} x + 2y - z &= 8 \\ 2x + 6y + z &= 12 \\ x + 3y + 4z &= -1 \end{aligned}$$

13. Solve by using the inverse of the coefficient matrix:

$$\begin{aligned} 2x - 3y &= 4 \\ x + 5y &= 2 \end{aligned}$$

14. The table below gives the interaction between two sectors in a hypothetical economy.

	Industry P	Industry Q	Final Demand	Total Output
Industry P	350	120	30	500
Industry Q	100	120	80	300
Labor	50	60		

Suppose that it is predicted that in 5 years the final demand for P will be 30 units, and that for Q will decrease to 70 units.

   (a) Write the system of equations which must be satisfied by the new total outputs for industries P and Q.

   (b) Find the new total outputs for each industry.

   (c) What will be the new labor requirements for each industry?

15. Solve this linear programming problem by the method of corners: Maximize $p = 200x + 160y$ subject to

$$\begin{aligned} 2x + 4y &\leq 70 \\ 5x + 3y &\leq 105 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

16. (a) Write a matrix equation that is equivalent to the given system of linear equations.
(b) Solve the system by use of the inverse of the coefficient matrix.
(NO CREDIT if solved by any other method)

$$\begin{aligned} x - y + 3z &= 2 \\ 2x + y + 2z &= 2 \\ -2x - 2y + z &= 3 \end{aligned}$$

Answers

1. \$30 or \$70
2. (a) C = 4x + 600 (b) 100
3. V(-1,4), concave down, points (-3, 0), (-2,3), (0, 3), (1,0)
4. (a) [3, infinity) (b) √ [(-3x - 2) / (x + 1)] (c) 1 / [√ (x - 3) + 1]
5. (a) -1, 3 (b) 190/53 (c) x = ln3
6. \$1,931,09
7. (a) ln 8/ ln 3 , or ~1.8928 (b) 15
8. ~ 5.6498 years
9. (a) ~ \$44.63 (b) ~ \$300
10. $x \leq -7$ or $x \geq 3$

11.

$$\begin{bmatrix} 15 & -24 \\ -12 & 21 \end{bmatrix}, \begin{bmatrix} 7 & -9 \\ -8 & 7 \end{bmatrix}, \begin{bmatrix} 42 & -5 \\ -36 & 4 \end{bmatrix}, \frac{1}{3}\begin{bmatrix} 7 & 8 \\ 4 & 5 \end{bmatrix}, \frac{1}{11}\begin{bmatrix} -1 & 2 & 6 \\ -2 & 4 & 1 \\ 6 & -1 & -3 \end{bmatrix}$$

12. x = 4, y = 1, z = -2
13. x = 2, y = 0
14. Equations: $x_1 = 0.7x_1 + 0.4x_2 + 30$, $x_2 = 0.2x_1 + 0.4x_2 + 70$
Outputs: 460 for P, 270 for Q. Labour: 46 for P, 54 for Q
15. x = 15, y = 10. $P_{max} = 4600$

16. a. $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$; b. $X = A^{-1}B = \begin{bmatrix} 1 & -1 & -1 \\ -\frac{6}{5} & \frac{7}{5} & \frac{4}{5} \\ -\frac{2}{5} & \frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ \frac{14}{5} \\ \frac{13}{5} \end{bmatrix}$

MATH 129 Supplementary Problems

1. Simplify these expressions:

   (a) $(2^{-1}x^3)^{-1}(3x^{-3})^2$ (b) $\left(\frac{(2x^2)(5x^3y)}{4x^2y^2}\right)\left(\frac{10xy^5}{3x}\right)$ (c) $(8x^6)^{2/3}$

   (d) $(x^{-1}+y^{-1})^{-1}$ (e) $\frac{x}{\sqrt{x-y}} - \sqrt{x-y}$ (f) $\frac{\sqrt{x}-2}{\sqrt{x}}$

2. Perform the indicated operations and simplify:

   (a) $3x(2x+1) - 5(4x-3)$ (b) $\frac{4x}{x^2+5x} + \frac{2}{x^2+2x}$

   (c) $\frac{x}{x^2+x-2} - \frac{2x-1}{2x^2+3x-2}$

3. Factor completely:

   (a) $25t^2 - 4$ (b) $12x^3 - 6x^2 - 18x$

4. Solve:

   (a) $x^2 - 8x + 3 = 0$ by using the process of completing the square.

   (b) $6x^2 = 7x + 4$ (any method)

   (c) $\sqrt{3t+1} = 5$ (d) $\frac{x}{3} - 1 = \frac{x}{4} + 1$ (e) $-4 \le 2 - x < 5$

   (e) $\frac{1}{3}[2 - 3(x+2)] = \frac{1}{4}\left[(-3x+1) + \frac{1}{2}x\right]$ (f) $x^2 - 2x \ge 8$

5. Find the equation of the line through the points $A(4,-5)$ and $B(6,1)$. Also find the equation of the line through the point A which is perpendicular to the line whose equation you just found. Find the x-intercepts and the y-intercepts for both these lines.

6. Given the functions $f(x) = 3x^2 + 5$ and $g(x) = \sqrt{2x+3}$, find:

   (a) $g(11)$ (b) $f(t-1)$

   (c) the domain of f (d) the domain of g

   (e) $\frac{f(x+h) - f(x)}{h}$

   (f) the composite function $f \circ g(x)$ (g) the composite function $g \circ f(x)$

7. A manufacturing company bought a new machine for \$60,000. This machine is to be depreciated linearly over 20 years with a scrap value of \$0. What will be the book value of the machine after 8 years?

8. A manufacturer has a monthly fixed cost of \$12,000 and a production cost of \$15 for each unit produced. The product sells for \$18 per unit. Let x represent the number of units.

   (a) What is the (total) cost function?

   (b) What is the revenue function?

   (c) What is the profit function?

   (d) How many units must be sold each month to break even?

Y

9. For the parabola with equation $y = f(x) = 4x^2 - 12x + 5$:

(a) Use the method of completing the square to write this equation in the form $y = a(x-h)^2 + k$.

(b) Find the coordinates of the vertex.

(c) Find the coordinates of the x-intercepts.

(d) Find the equation of the axis of symmetry.

(e) Sketch the graph of $y = f(x)$. Include and label the vertex and the intercepts.

10. The monthly revenue for a certain product is given by the equation $R(p) = 50p - 0.2p^2$ where p is the selling price in dollars and R is the revenue in dollars. Find the price which should be charged in order to maximize revenue. Why does this price yield a maximum (rather than a minimum) revenue?

11. Simplify:

(a) $\ln\left[\frac{(x+1)^4}{x^2+5}\right]$ (b) $\ln\left(x^2 e^x\right)$

12. Solve for x without using a calculator. Check your answers.

(a) $2^{(x+1)} = \frac{1}{16}$ (b) $\log_3 x = -2$

(c) $e^{(3x-1)} = 5$ (d) $\ln(x) + \ln(x+2) = \ln(8)$

13. Solve for x using calculator. Give answer to 4 decimal places

(a) $3(5^x) = 24$ (b) $x = \log_3 6$

(c) $\frac{8}{2+e^{-x}} = 3$ (d) $3e^{(2-3x)} = 19$

14. Solve for x:

(a) $\log_{(1/8)}(27) = x$ (b) $\log_x(125) = 3$

(c) $\log_{12}(x-2) = 2$ (d) $\log_5(10) = x$

(e) $e^{2\ln(x+1)} = 16$ (f) $\log_3 x + \log_3(2x+51) = 4$

(g) $\log(x+2) - \log(x-1) = \log 4$

(h) $\ln(x+4) - \ln(x-4) = 2\ln 3$

(i) $5^{2x+1} = 30$ (k) $\frac{10}{1+2e^{-x}} = 5$

(l) $(2^x)^x = 25$ (m) $5e^{2x-1} = 32$

15. A company purchased a new machine for \$100,000. After 3 years its resale value has decreased to \$85,000.

(a) Assume that the value is decreasing by the same amount every year.

(i) Find the equation for $V(t)$, the value of the machine t years after it was purchased.

(ii) What will the resale value be 10 years after the machine was purchased?

(b) Assume that the value is decreasing by the same percent every year.

(i) Find the equation for $V(t)$, the value of the machine t years after it was purchased.

(ii) What will the resale value be 10 years after the machine was purchased?

(iii) By what percent does the value decrease each year?

16. The decay constant for carbon 14 is $k = 0.00012$.

(a) Wood deposits recovered from an archeological site contain 35% of the carbon 14 they originally contained. How long ago did the tree from which the wood was obtained die?

(b) What is the half-life of carbon 14?

17. Initially, 400 rabbits were released in a park. In 10 years, the rabbit population had grown to 600. Assume the rabbit population is increasing exponentially.

(a) Write an equation giving the population of rabbits of the park in terms of the number of years, t, since the 400 rabbits were released.

(b) If the rabbit population keeps increasing at this rate, how long will it take for the population to grow to 800 rabbits?

18. Sketch the graphs of these functions:

(a) $y = f(x) = 3^x$ (b) $y = g(x) = 3^{-x}$ (c) $y = h(x) = \log_3 x$

19. Which of the following investments is best for the investor. Show work to back up your answer.

(a) nominal annual rate of 2.0% compounded quarterly.

(b) nominal annual rate of 1.98% compounded continuously.

20. The sum of \$1000 is invested at a nominal annual rate of 3% compounded quarterly. How long will it take for it to grow to \$1500?

21. How long will it take for the sum of \$1000 to grow to \$1500 if it is invested at the rate of 3% compounded continuously?

22. A person deposits \$200 at the end of every month into a savings account which pays interest at a nominal annual rate of 2.4% compounded monthly. Interest calculations are made at the end of each month. If he doesn't take any of this money out, how much will he have after 5 years?

23. A loan of \$20,000 is to be paid back through equal monthly payments over a period of 15 years. The nominal annual interest rate is 5.4%. Interest calculations are made at the end of each month. How much will each payment be?

24. Solve the following system by using aumented matrices and row-reduction.

$$\begin{aligned} x + y + z &= 2 \\ x + 3y + 3z &= 4 \\ 2x + y + 3z &= 7 \end{aligned}$$

25. Let $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ 8 & -1 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 0 & 2 \\ 4 & 1 & 2 \end{bmatrix}$, and $D = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 1 & 6 \end{bmatrix}$. Compute the following. (If any of the quantities is not defined, the answer to that part is NOT DEFINED.)

(a) $3B$ (b) $A + B$ (c) AC (d) CA

(e) A^{-1} by the row-reduction method.

(f) D^{-1} by the row-reduction method.

26. Given that each of the following row-reduced augmented matrices is equivalent to the augmented matrix of a system of linear equations in the variables x, y, and z, first determine whether the system has any solution .If it doesn't, indicate why. If it does, find the solution or solutions to the system.

(a) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

27. The table below gives the interaction between two sectors in a hypothetical economy. Assume that this economy satisfies the assumptions for input-output analysis.

	Industry P	Industry Q	Final Demand	Total Output
Industry P	140	40	20	200
Industry Q	20	20	60	100
Labor	40	40		

Suppose that it is predicted that in 3 years the final demand for P will be 30 units and that for Q will be 60 units.

(a) Write the system of equations which must be solved to find the total outputs for P and Q in 3 years.

(b) What is the input-output matrix for this economy?

(c) Use matrix methods to find what each industry's total output should be to meet the predicted demands for 3 years from now.

(d) What will the labor inputs be for industry P in 3 years?

28. Find the maximum and the minimum values of $z = x - y$ subject to these constraints:

$$\begin{aligned} 3x + 2y &\leq 7 \\ 2x + 5y &\leq 12 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$