Factoring a polynomial over the integers, in one variable:

Perform these steps repeatedly in order until you are done with each step.

1. **Simplify** by combining like terms; then write in **descending order.**
2. Factor out the **greatest common monomial factor**.
3. **Difference of two squares**.
4. **Difference of two like odd powers**: $a^5-b^5 = (a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$
5. **Sum of two like odd powers**: $a^7+b^7 = (a+b)(a^6-a^5b+a^4b^2-a^3b^3+a^2b^4-ab^5+b^6)$
   FOR BOTH #4 and #5: if sign in first factor is +, signs in second factor alternate; else, all signs in 2nd factor are +. The total degree of each term in the second factor is the same.
6. **Trinomial: ax^2+bx+c = (maybe) $(dx+e)(fx+g)$**
   a. If needed, factor out a (-1) so that the term of highest degree is positive.
   b. If **c is positive** then e and g both have the **same sign**.
      i. Then, if b is positive, e and g are both positive.
      ii. Or, if b is negative, e and g are both negative.
   c. If c is negative, then e and g have opposite signs.
   d. Write all possible factor pairs of a; these are possible values of d and f.
   e. Write all possible factor pairs of c; these are possible values of e and g.
   f. Try all possible combinations of (dx+e)(fx+g). If they fail, there is no factorization over the integers.
   g. Use Dr. L's divisibility trick. If you already did step (2) completely, then it is not possible for d and e to have a common factor; and it is not possible for f and g to have a common factor. This is true because any common factor of (say) d and e would also be a factor of every term of the original trinomial. This way, you can sometimes reject candidate factors.
   h. To shortcut your work, use the discriminant $\Delta = b^2 -4ac$ of the trinomial. The trinomial factors over the integers iff Δ is a (positive) perfect square.
7. Further work on trinomials: You can solve the equation **$ax^2+bx+c=0$** by using the **quadratic formula.** If r1 and r2 are the roots of that equation, then (x-r1) and (y-r2) are factors [but you might have to multiply these factors by some integers to get the correct answer!
8. #7 is a special case of the Factor Theorem: if P(x) is any polynomial in x, then:
   a. (x-r) is a factor of P(x) iff P(r) = 0.
   b. A corollary of the Factor Theorem is the Rational Root theorem (not shown here).
9. Special Products and Grouping. Examples of these are omitted here.

Factoring a polynomial in two variables:

10. **$ax^2+bxy+cy^2$ may be factored as follows:**
    a. **Set y=1. Then factor ax^2+bx+c (if possible) as** (dx+e)(fx+g).
    b. **Then the factors of $ax^2+bxy+cy^2$ factors as:** (dx+ey)(fx+gy).
    c. **Example:** $4x^2+4xy+y^2$. Then $4x^2+4x+1 = (2x+1)(2x+1)$. So, $4x^2+4xy+y^2 = (2x+y)(2x+y)$.

Factoring a polynomial (of higher degree) over $\mathbb{R}$ (reals) or $\mathbb{C}$ (complex numbers):

11. Sketch the polynomial, picking points wisely. Find the critical points if possible. Use the fundamental theorem of algebra. And, if the polynomial is positive at $x=x_1$ and negative at $x=x_2$, then it must take the value zero somewhere between x_1 and x_2. As you find factors, use polynomial long division to reduce the degree of the polynomial.

How to factor (almost) anything. Name________________________ Section _______________

Directions: Factor completely over the rational numbers. Check by multiplying! If the expression cannot be factored, write "prime". Do these problems in your HW book. Copy answers here.

	Problem	Work	Answer
1	x^2-9		
2	$4x^2-25$		
3	$49-9x^2$		
4	$1-169x^2$		
5	$3x^3-27x$		
6	$5x^4 -625x$		
7	$(x+7)^2-16$		
8	x^5-32x		
9	x^2+12		
10	x^2-11		
11	$x^2-3x+6-2x$		
12	$x^2-9x+20$		
13	$6w^2-7w-20$		
14	$5wx^2-110wx+605w$		
15	x^3+y^3		
16	x^3-8		
17	$5x^6y^6 -625x^3y^3$		
18	x^2-5x+6		
19	$6x^2+11x-10$		
20	x^2+x+3		
21	x^3-49x^2-343x		
22	$8x^4+64x$		
23	$8x^4+81x$		
24	$(2x+1)^2 -3(2x+1)+2$ [Hint: let $p = (2x+1)$]		
25	$6x^2-5x-6$		
26	$6x^2-7x-6$		
27	$5x^2-35x+65$		
28	w^9-x^3		
29	$16x^2-8xy+y^2$		
30	x^6-y^6		
31	$X^4-3x^3-x^2+3x$ Hint: $(x-3)$		
32	$x^4-4x^3-x^2+16x-12$ Hint: factor theorem		