Amortization and sinking funds.   Name _____

I.   Amortization of a loan over time.

Example: home mortgage (fixed rate).
Interest is charged at a fixed rate on the unpaid portion of the debt (the principal). The payment amount is a fixed periodic payment (usually, monthly).
  Specific example: home mortgage, fixed interest of 5%, 30-year term, equal monthly payments.
Fact: at the beginning of the loan period, almost the entire payment is interest. The principal reduces very slowly at the start of the loan. Near the end of the loan period, almost the entire monthly payment is principal; the principal reduces quickly.

Say, P = original amount borrowed = the principal. Let r = the annual interest rate. Let m = the number of payments per year (often, n=12).   Let i = interest rate PER PERIODIC PAYMENT (Then i = r/m). Let n = the total number of periodic payments. The periodic payment R is given by:

$$R = \frac{P\,i}{1 - (1 + i)^{-n}}$$

II.   Sinking funds

A sinking fund is an account set up for a specific purpose at some future date. E.g. a company might set up a sinking fund to accumulate enough capital to replace equipment that is expected to be obsolete at a future date.

We can think of the amount to be accumulated by a specific date in the future as the FUTURE VALUE of the annuity.

Say, S = the future value. Let i = interest rate earned in each conversion period. Let   r = the annual interest rate. Let n = number of periodic payments. Let m = the number of payments per year. (Then, i= r/m). The periodic payment R needed to accumulate the future value of S dollars over n periods with interest charged at the rate of   i per period is given by:

$$R = \frac{S\,i}{(1 + i)^{n} - 1}$$

Problems.
1.   Cindy borrows $100,000 for a home mortgage at a rate of 6% , at 12 payments per year, for a period of 30 years.
     a.   Find Cindy's periodic payment.
     b.   What is the total amount Cindy pays to the mortgage lender over the 30 year period. How does that compare to the original amount of $100,000 ?
2.   Mr. Yang sets up a sinking fund to accumulate the future value of $25,000 in a 5-year period, to replace a large refrigerator in his butcher shop. He wants to make 4 payments per year. The bank offers him a 3% annual interest rate on the sinking fund.
     a.   How many payments will he need to make?
     b.   What is the interest rate in each conversion period?
     c.   What will Mr. Yang's quarterly payment need to be?