Solving Absolute Value Equations and Inequalities.

Basic concept: the **geometric distance** from x to y is the POSITIVE distance between x and y.

Absolute value notation. $|x| \equiv$ "the <u>absolute value</u> of x".

Algebraic definition: $|\mathbf{X}| = \begin{cases} x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0. \end{cases}$

Geometric definition #1: |x| = The geometric distance from x to 0. (Note that "geometric distance" is never negative!) Geometric definition #2: |x-y| = The geometric distance from x to y.

Examples

	Problem	Work	Answer
1	Solve x = 3	Put 0 on a number line. What numbers x are 3	{-3,3}
		units away from 0? Answer: -3 and 3.	
2	Solve x-2 = 3	Put 2 on a number line. What numbers x are 3	{-1,5}
		units away from 0? Answer: -1 and 5.	
3	Solve x-2 = 3	Think of "2" as the center, and "3" as the radius,	{-1,5}
	(second method)	of a "circle". On the number line, it looks like	
		this:	
		$\overleftarrow{} 3 \rightarrow \overleftarrow{} 3 \rightarrow$	
		-1 2 5	
4	Solve x-2 < 3	Put 2 on a number line. What numbers x are	-1 < x < 5 (formula answer)
		less than 3 units away from 0? Answer: any	(-1,5) (interval answer)
		number between -1 and 5.	
			-1 2 5 (graph answer)
5	Solve $ x-2 \le 3$	Put 2 on a number line. What numbers x are no	$-1 \le x \le 5$ (formula answer)
		more than 3 units away from 0? Answer: any	[-1,5] (interval answer)
		number between -1 and 5, including -1 and 5.	
			-1 2 5 (graph answer)
6	Solve $ x-2 \ge 3$	Put 2 on a number line. What numbers x are no	$x \le -1$ or $5 \le x$ (formula answer)
		less than 3 units away from 0?	$(-\infty, -1] \cup [5, \infty)$ (union of
		Think: the distance from x to 2 MUST BE \geq 3.	intervals)
		The x values are all OUTSIDE OF the interval	$\longleftrightarrow \bullet \rightarrow$
		(-1,5).	
		and 5.	(graph answer)
7	Solve 3x+5 >2	In order to use the graphical method as in #5	Graph answer:
		above, we convert the number inside the	intervals)
		absolute value to a difference.	
		3x-(-5) > 2.	-5/3-2/3 -5/3+2/3
		Next, factor the coefficient 3 from the inside:	Or
		3(x-(-5/3)) > 2. Bring the 3 outside the .	
		3 x-(-5/3) > 2. Now divide both sides by 3.	-7/3 -3/3 = -1
		x-(-5/3) > 2/3. Use -5/3 as the center, 2/3 as	Interval answer:
		the radius.	(-∞,-7/3] ∪ [−1, ∞)