# Sample of Typical Final Examination Problems

Math 130 Precalculus for the May 20, 2016 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. Find an equation of the line passing through the points $(8, -1)$ and $(-2, 3)$. Sketch the line and label both intercepts with their coordinates.

2. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\dfrac{f(x+h) - f(x)}{h}$ and simplify the result.

3. For $f(x) = \sqrt{x}$ and $g(x) = x^4 - 6x^2 + 9$, find and simplify $f \circ g$ and $g \circ f$. The simplified results will have no square roots.

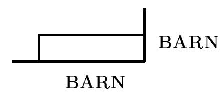
   Find the domain of each function and the domain of each composite function (over the real numbers).

4. Determine whether $f(x) = \dfrac{4}{-5x+3}$ has an inverse function. If it does, find the inverse function.

5. Write the quadratic function $f(x) = \frac{1}{4}x^2 - 2x - 12$ in standard form and sketch its graph.

   Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.

6. Find the largest area that a farmer can enclose by constructing a rectangular pen from 26 feet of fencing, if he uses a corner of his barn for two walls of the pen.




7. Let $w = \log_2 a$.

   Find an expression in terms of $w$ for:

   (a) $\log_2 a^4$
   (b) $\log_2(4a^2)$
   (c) $\log_2(4a)^2$
   (d) $\log_2(\sqrt[4]{a})$
   (e) $[\log_2 4a]^2$
   (f) $\sqrt{\log_2 a^2}$

8. In each of parts (a) through (h), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 1.098$, $\log_b 3 \approx 1.740$, and $\log_b 5 \approx 2.5495$.

   (a) $\log_b 6$
   (b) $\log_b \frac{3}{5}$
   (c) $\log_b 125$
   (d) $\log_b \sqrt{3}$
   (e) $\log_b 20$
   (f) $\log_b(4b)^{-2}$
   (g) $\log_b(5b^2)$
   (h) $\log_b \sqrt[3]{2b}$

9. (a) Simplify to an exact rational number: $\log_2\left(\sqrt[3]{128}\right)$.
   (b) How is $\log_2 64a^2$ related to $\log_2 a$?

10. Solve algebraically: $\log_2 x + \log_2(1 - 3x) = -4$.

    Hint: to solve the equation found after applying some log rules, just use the quadratic formula.

11. First decide in which intervals all valid solutions must lie.

    Then solve for $x$.

$$\log_2 x + \log_2(1 - 2x) = -3.$$

    Check your solutions in the original equation.

12. Decide if each statement is true or false. Then justify your answer by writing an equation.

    (a) Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.
    (b) Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.

13. Prove without a calculator that $\log_4 7$ is just about equal to 1.4.

Hint: when does $4^{m/n}$ come to close to 7? Start with lists of powers of 4 and of powers of 7. Hand multiply to find the first seven members of each list.

14. (a) Rewrite in radian measure as a fractional multiple of $\pi$ and in degree measure: 3/16 of a revolution.
   (b) Rewrite in degree measure: $\dfrac{7\pi}{8}$.
   (c) Rewrite in radian measure as a fractional multiple of $\pi$ in lowest terms: 132°.
   (d) Find the length of the arc on a circle of radius $150/\pi$ feet intercepted by a central angle of 150°.

15. A right triangle has an acute angle $\theta$ with $\sec\theta = \frac{8}{7}$. Find the exact values of the other five trigonometric functions of $\theta$, in fractional form. Some of the expressions will involve square roots; do not converst the square roots to decimals.

Then find the exact values of $\sec(90° - \theta)$ and of $\csc^2\theta - 1$, also in fractional form.

*Hint.* First sketch a right triangle corresponding to that secant. Next use the Pythagorean Theorem to determine the third side. Then find the other five trigonometric functions of $\theta$.

For the other two values, use the appropriate trigonometric identities.

16. Given that $\cos u = -4/5$ with $\pi/2 < u < \pi$, find the exact values of $\sin 3u$ and $\cos 3u$ using the double-angle formulas and the sum formulas, as needed. Don't use a calculator. *Hint:* use fractions, not decimals.

17. Simplify and reduce to an expression that contains at most one trig function.
   (a) $\cos x(1 + \tan x)(1 - \tan x)$
   (b) $\tan x \cos^2 x$
   (c) $\cos^4 x - \sin^4 x$
   (d) $\dfrac{1 + \cot^2 x}{\sin x}$
   (e) $\dfrac{\sec x}{\csc x}$
   (f) $\dfrac{\sec x}{\sin x}$

18. Find all solutions of $3\cos 2x = 2\cos^2 x$ in the interval $[0, 2\pi)$. (Get the algebraically-assisted, exact result.)

19. Use the Law of Cosines to solve the triangle with sides of lengths 1 and $\sqrt{3}$, and an included angle of 30° between them.

20. Use the Law of Cosines to solve the triangle with sides of lengths 2 and $5\sqrt{3}$, and an included angle of 30° between them. (Round angles to three decimal places.)

*Hint.* After finding the third side ask yourself: Which is the longest side? So the largest angle will be between which two sides? Which is the shortest side? So the smallest angle will be between which two sides?

Find the exact area of this triangle. Leave it as multiple of $\sqrt{3}$.

*Hint.* Drop a perpendicular from the opposite vertex to the side of length $5\sqrt{3}$.

21. (a) A triangle has sides of lengths 5 inches and 8 inches, with an angle of 30 degrees between them. Find the exact area of the triangle. No calculator is needed. Start by drawing the triangle.
   (b) A triangle has sides of lengths 5 inches and 8 inches, with an angle of 150 degrees between them. Find the exact area of the triangle. No calculator is needed. Start by drawing the triangle.
   (c) An isosceles triangle has an angle of 120 degrees and two equal sides of 1 inch. Solve this triangle. No calculator is needed. Start by drawing the triangle. The Law of Cosines would help in solving the triangle.
   (d) Find the area of a right triangle with hypotenuse equal to 1 and an angle of $\pi/6$.

22. Find the period and the amplitude of

$$y = 5\sin\left(2x - \frac{\pi}{4}\right).$$

Graph one period.

It might be less confusing with the 2 factored out of the expression in the parentheses.