Factoring natural numbers: Notes . Name ________________________________

1. Definition: a natural number x is a **factor** of the natural number y, iff there is a natural number c, with xc=y.
2. Definition: a natural number is a **prime number**, iff it has exactly two factors, itself and 1.
3. Remark: The number 1 is NOT a prime number. (Why?)
4. Here is a list of the first few prime numbers:

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71

5. Divisibility tricks: Divisible by 2 if last digit is even. Divisible by 3 if digits sum to a multiple of 3. No trick exists for 7. Trick for 11 is more complicated. Otherwise, you must divide.
6. Theorem (Euclid): The number of prime numbers is infinite.

Proof: Suppose not. Then let Q = the product of all possible prime numbers. Consider Q+1. Q+1 cannot be divisible by any of the prime numbers, because the remainder when dividing by that prime number would be 1 (not zero). Therefore, either there is another prime not in the list of primes, that is less than Q+1, that is a factor of Q+1; or else Q+1 is itself prime. Either way, the original list of primes was not complete. So there cannot be a finite number of primes.

7. Definition: If a natural number C is not prime, we say it **can be factored**.
8. Remark: suppose AB=C. Suppose $A \leq B$. Then, either $A < B$ or $A = B$.
9. Theorem: Suppose C is a natural number. If C can be factored, then C has a factor A with $A^2 \leq C$.

Proof: follows from the remark above.

10. Algorithm: **How to factor a natural number into prime factors**:

If C is a natural number, then we should try to divide C by each prime number less than or equal to the square root of C. If one of those primes is a factor, then C can be factored. Suppose p_1 is that factor. Let $C_1 = C/p_1$. Then continue as before, using C_1 in place of C. Continue until the result is prime. Write the result as a product of primes in ascending order, with exponents.

11. Example: Factor 323 into prime factors.

We need to try the primes 2,3,5,7,11,13,17. Since $18^2 = 324$ (bigger than 323), we don't have to try any primes bigger than 17. After we divide each of these into 323, we find that 17(19)=323. Therefore $323 = 17 \cdot 19$.

12. Example: Factor 96 into prime factors.

96/2 = 48. 48/2 = 24. 24/2 = 12. 12/2 = 6. 6/2 = 3. Therefore, $96 = 2^5 \cdot 3$.

13. Example: Factor 331 into prime factors.

$19^2 = 361$ (greater than 329), so we don't need to try any primes bigger than 17.
When we try the primes 2,3,5,7,11,13,17, we find that there is a non-zero remainder for each division. Therefore, 331 is a prime number.

14. Exercise: Factor these numbers into prime factors if possible:
12, 28, 64, 100, 132, 327, 441, 1058, 1728.