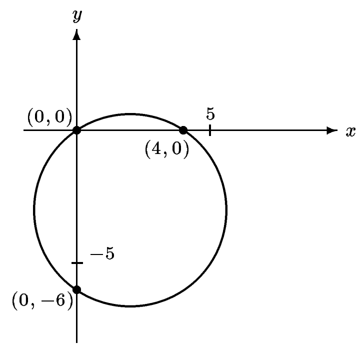
Answers to the Sample of Typical Final Examination Problems

Math 130 Precalculus for the December 20, 2013 Final Exam

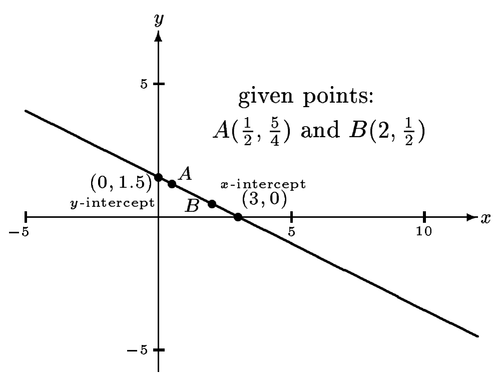
1. The point is $(17, -4)$. A line segment connecting any other point on that line to either A or B has the same slope, $-3/5$, as does AB.

   But in this case if B is equally as far away from A and C and on the same line, it must be the midpoint of the line segment AC. So set $B(7, 2)$ to the result of finding the midpoint between $(-3, 8)$ and (x, y). Then solve the equations $7 = \dfrac{-3 + x}{2}$ and $2 = \dfrac{8 + y}{2}$ to get $(17, -4)$.

2. $(x - 2)^2 + (y + 3)^2 = 13$. The center is at $(2, -3)$ and the radius is $\sqrt{13}$.



3. $y = -\frac{1}{2}x + \frac{3}{2}$ or $y - \frac{1}{2} = -\frac{1}{2}(x - 2)$ or $x + 2y = 3$.



4. $$\frac{f(x+h) - f(x)}{h} = \frac{(2x^2 + 4xh + 2h^2 + 3x + 3h - 1) - (2x^2 + 3x - 1)}{h} = \frac{4xh + 2h^2 + 3h}{h} = 4x + 2h + 3,\ h \neq 0.$$

5. Reflection in the x-axis, vertical shrink by a factor of one-half, and vertical shift two units down of the parent function $y = \sqrt{x}$. The function shown in the graph has equation $y = -\frac{1}{2}\sqrt{-x} - 2$.

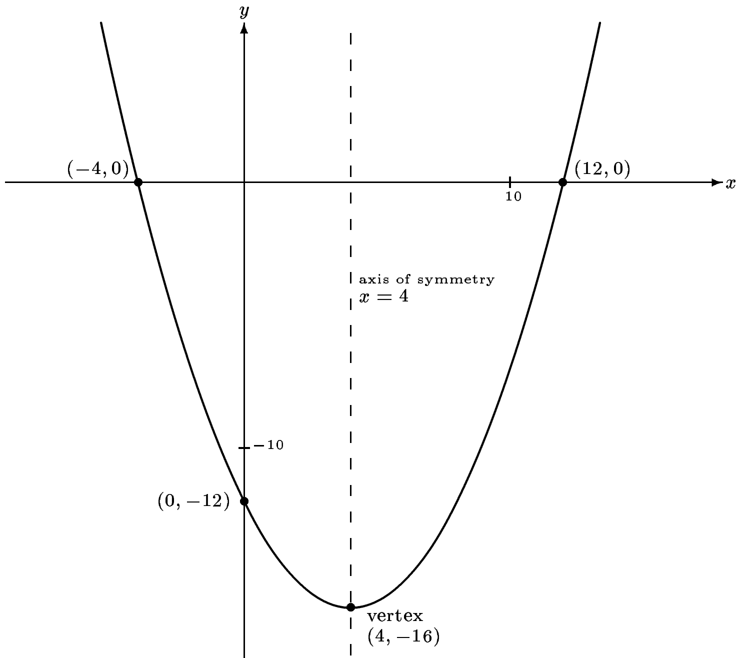
6. (a) $\sqrt{x^2 + 4}$ (b) $x + 4$

   Domains of f and $g \circ f$: all real numbers x such that $x \geq -4$

   Domains of g and $f \circ g$: all real numbers

7. Yes; $h^{-1}(x) = \dfrac{3 - x}{5}$.

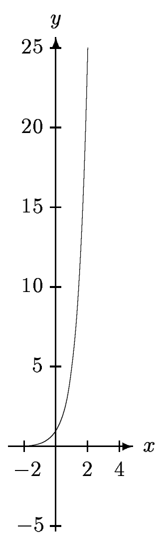
8. $f(x) = \frac{1}{4}(x-4)^2 - 16.$

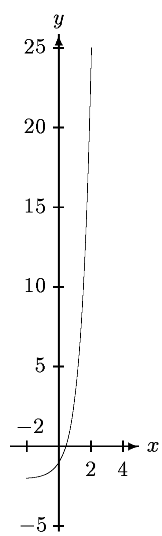


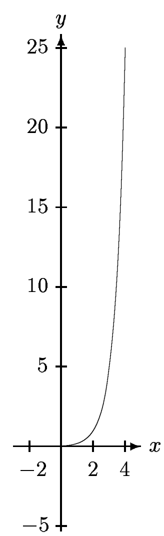
9. 21, 7

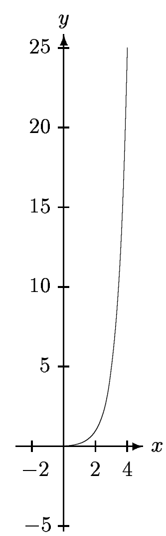
10. Original equation. (a) Verical shift two units down. (b) Horizontal shift two units to the right. (c) Vertical shrink of one twenty-fifth.

This is $f(x)$. This is $f(x)-2$. This is $f(x-2)$. It's $cf(x)$: $c = \frac{1}{25}$ and $0 < c < 1$.









By inspection, graphs (b) and (c) look the same. Algebraically: $5^{x-2} = 5^x/5^2 = 5^x/25 = \left(\frac{1}{25}\right)5^x$.

Take three points on the original graph and transform them: The original graph, $y = 5^x$, has points $(-2, 0.04)$, $(0, 1)$, and $(2, 25)$. Graph (a) has points $(-2, -1.96)$, $(0, -1)$, and $(2, 23)$. Graph (b) has points $(0, 0.04)$, $(2, 1)$ and $(4, 25)$. Graph (c) has points $(-2, 0.0016)$, $(0, 0.04)$, and $(2, 1)$.

Since both the original equation and equation (a) have a point on the graph with $x = 0$ that differs from the others, they cannot be equivalent to any of these equations. But equations (b) and (c), so far, seem to have the same graph. The definition of equivalent equations is that they have the same solution set and the same graph. So, just by looking at the points alone without the algebraic equivalence, it appears that (b) and (c) are equivalent equations.

11. all of them

12. (a) $4w$ (b) $2w+2$ (c) $2w+4$ (d) $w/4$ (e) w^2+4w+4 (f) $\sqrt{2w}$

13. (a) 10/3 (b) 49/16 or 3.0625

14. 125/256

15. (a) 2.838 (b) -0.8095 (c) 7.6485 (d) 0.87 (e) 4.7455 (f) -6.392 (g) 4.5495 (h) 0.69933

16. Yes. it can be determined that $\log_b 10 \approx 2.9251$

17. (a) $x = 20$ (b) $x = 500/33$ or $x = 15\frac{5}{33}$ (c) $x = 7$ (d) $x = 25$

18. (a) True.
    $\log_a uv = \log_a u + \log_a v$
    This is the first prpoerty of logarithms.
    (b) False.
    $(\log_a u) \div (\log_a v) = \log_a (u - v)$
    $2 = \log 100 \div \log 10 \neq \log(100 - 10) = \log 90 \approx 1.95$

19. About 566 bacteria

20. (a) (i) $\frac{13\pi}{18}$ (ii) $-\frac{\pi}{3}$

(b) (i) 270° (ii) 225°

(c) 2.5π meters ≈ 7.85398 meters

21. (a) 8π rad/min ≈ 25.1327 rad/min (b) 628.31853 ft/min (c) 7.14 miles

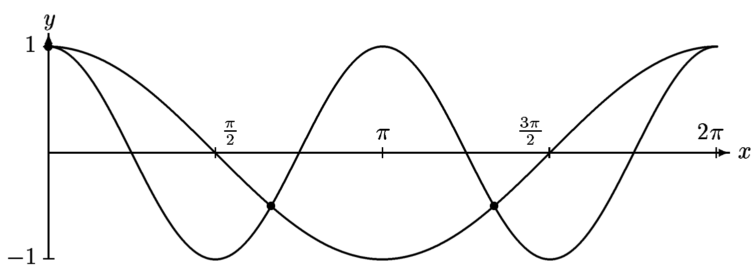
22. $\sin\theta = \frac{\sqrt{5}}{3}$ $\cos\theta = \frac{2}{3}$ $\tan\theta = \frac{\sqrt{5}}{2}$ $\csc\theta = \frac{3\sqrt{5}}{5}$ $\cot\theta = \frac{2\sqrt{5}}{5}$

23. (a) $\cos x$ (b) -1 (c) $2\cot^2 x + 1$ (d) $\sec^2 x$ (e) $-\cos^2 x$ (f) $\sin^2 x$

24. $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

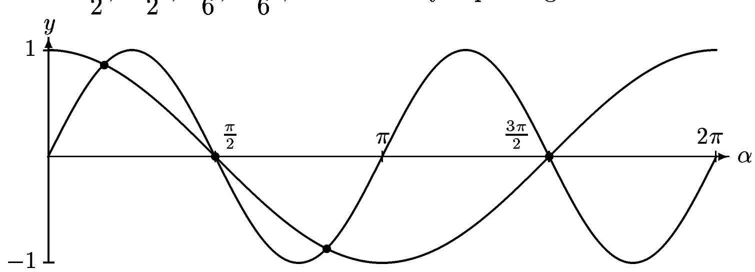
25. $\sin^{-1} 0.6$ or 36.8699° or 0.6435 radians

26. (a) $0, \frac{2\pi}{3}, \frac{4\pi}{3}$; obtained by replacing $\cos 2x$ with $2\cos^2 x - 1$ and solving for x.



The two graphs indeed look like they intersect when $x = 0$, $x = \frac{2}{3}\pi$, and $x = 1\frac{1}{3}\pi$. The apparent fourth point, at $x = 2\pi$, is not in the interval given, as it is just a one-period shift on the cosine graph of the point when $x = 0$.

(b) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$; obtained by replacing $\sin 2\alpha$ with $2\sin\alpha\cos\alpha$ and solving for α.



The two graphs obviously intersect when $\alpha = \frac{\pi}{2}$ and when $\alpha = \frac{3\pi}{2}$. The other two points are clearly between 0 and $\frac{\pi}{4}$, and between $\frac{3\pi}{4}$ and π, so the actual values are reaonable guesses.

27. $\sin\frac{11\pi}{12} = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$; $\cos\frac{11\pi}{12} = -\frac{1}{4}\left(\sqrt{6} + \sqrt{2}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$; $\tan\frac{11\pi}{12} = -2 + \sqrt{3}$
It doesn't matter; either method you choose to break up the angle yields the same result.

28. $\sin 2u = -\frac{24}{25}$, $\cos 2u = \frac{7}{25}$, $\tan 2u = -\frac{24}{7}$

29. Each trig function will take on the same value in both cases, because the double angles will differ by 2π.

30. The third side is $\sqrt{127}$ (about 11.2694) long, the angle opposite the side with length 6 is 27.4571°, and the angle opposite the side with length 7 is 32.5429°. Note that these 3 angles add to 180.0000°.