Trig Practice for Final Exam
Math 130, Fall 2013

1. (a) Convert to radian measure as a multiple of π. (i) $-16°$ (ii) $72°$
   (b) Convert to degree measure:
      i. $\frac{4\pi}{15}$
      ii. $-\frac{5\pi}{12}$

   Both of these numbers are in radian measure.

2. How long is an arc associated with an angle of $12°$ in a circle with radius 4000 miles?

   Since the radius of the earth is about 3959 miles, this gives an approximation of the distance between two points on the equator whose longitudes differ by $12°$.

3. On a turntable, a 5-in. diameter record is rotating at a rate of 78 revolutions per minute (rpm).

   Find
   (a) the angular speed of the record;
   (b) the linear speed of a point on its rim.

4. A right triangle has an acute angle θ with $\csc\theta = \frac{41}{40}$. Find the exact values of the other five trigonometric functions of θ, in fractional form.

5. Match each trigonometric expression in (a)–(f) with one of the following.

   (A) $\sec x$ (B) -1 (C) 1 (D) $\tan x$ (E) $\csc^2 x$ (F) $1 + 2\cot^2 x$

   (a) $\csc x \sin x$ (b) $\tan^2 x - \sec^2 x$ (c) $\csc^4 x - \cot^4 x$ (d) $\tan x \csc x$ (e) $\dfrac{\csc^2 x - 1}{\cos^2 x}$ (f) $\dfrac{\cos[(\pi/2) - x]}{\cos x}$

6. Find all solutions of $2\cos^2 x = 1 + \sin x$ in the interval $[0, 2\pi)$. (Get the algebraically-assisted, exact result.)

7. Find the exact values of the sine, cosine, and tangent of $\dfrac{7\pi}{12}$. (No credit for decimal answer.)

8. Given that $\cos u = -4/5$ with $\pi < u < 3\pi/2$, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

9. Use the Law of Cosines to solve the triangle with sides of lengths 3 and 8, and an included angle of $60°$ between them. (Round angles to three decimal places.)

10. Sketch the graph of $\sin(x - \frac{2\pi}{3})$ for $0 \le x \le 2\pi$, labeling with coordinates all intercepts and the peak and valley (you may use the letter π and square root signs, as needed, in the labels—decimal equivalents are not required).

11. Sketch the graph of $\cos(x - \pi/6)$ for $0 \le x \le 2\pi$, labeling with coordinates all intercepts and the peak and valley (coordinates may be left in terms of π).

Answers follow.

Answers.

1. (a) (i) $-\frac{4}{45}\pi$ (ii) $\frac{2}{5}\pi = 0.4\pi$
   (b) i. $48°$
   ii. $-75°$

2. Using 4000 miles: about 838 miles. With the exact value of 3959 miles: 829 miles.
   For the approximation using 4000 miles, it could be done without a calculator with $\pi \approx 3.1416$.

3. (a) 2.6π radians per second.
   (b) 6.5π inches per second.

4. $\sin\theta = 40/41$
   $\cos\theta = 9/41$
   $\tan\theta = 40/9$
   $\sec\theta = 41/9$
   $\cot\theta = 9/40$

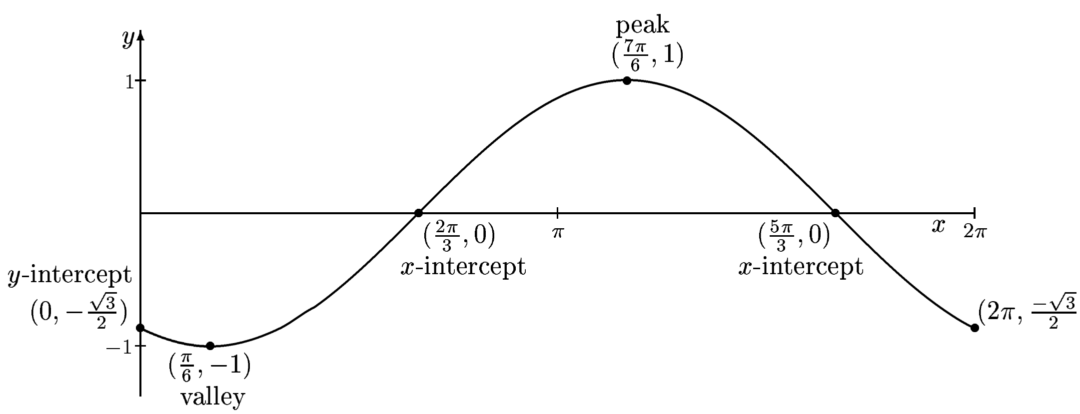
5. (A)(d); (B)(b); (C)(a); (D)(f); (E)(e); (F)(c)

6. $\pi/6$, $5\pi/6$, and π.

7. The sine is equal to $\frac{1}{4}\left(\sqrt{2}+\sqrt{6}\right)$.
   The cosine is equal to $\frac{1}{4}\left(\sqrt{2}-\sqrt{6}\right)$.
   The tangent is equal to $-\left(2+\sqrt{3}\right)$.

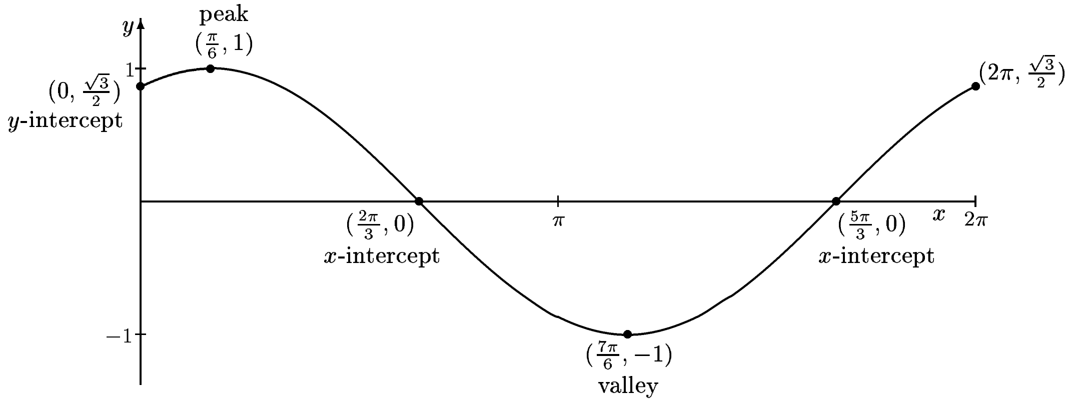
8. $\sin 2\theta = 2(-3/5)(-4/5) = 24/25$.
   $\cos 2\theta = 2(-4/5)^2 - 1 = 7/25$.
   $\tan 2\theta = \sin 2\theta / cos 2\theta = \frac{24/25}{7/25} = 24/7$.

9. The third side has length 7; the angle between the sides of lengths 3 and 7 is equal to $98.213°$; and the angle between the sides of lengths 7 and 8 is equal to $21.787°$.

10. 

   To get the valley, note that on the graph of $\sin x$ a valley is $(-\frac{\pi}{2}, -1)$. Then, to obtain the corresponding point on $\sin(x - \frac{2\pi}{3})$, add $\frac{2\pi}{3}$ to x, since the point is moved $\frac{2\pi}{3}$ to the right.
   The new valley is $(-\frac{\pi}{2} + \frac{2\pi}{3}, -1) = (\frac{\pi}{6}, -1)$.
   To get the y-intercept, let $x = 0$ in the equation $y = \sin(x - \frac{2\pi}{3})$, obtaining
   $\sin(0 - \frac{2\pi}{3}) = \sin(-\frac{2\pi}{3}) = -\sin(\frac{2\pi}{3}) = -\sin(\pi - \frac{\pi}{3}) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$.

11.



To get the valley, note that on the graph of $\cos x$ the valley is $(\pi, -1)$. Then, to obtain the corresponding point on $\cos(x - \frac{\pi}{6})$, one adds $\frac{\pi}{6}$ to x, since the point is moved $\frac{\pi}{6}$ to the right.

The new valley is $(\pi + \frac{\pi}{6}, -1) = (\frac{7\pi}{6}, -1)$.

To get the y-intercept, let $x = 0$ in the equation $y = \cos(x - \frac{\pi}{6})$, obtaining $\cos(0 - \frac{\pi}{6}) = \cos(-\frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$.