Algebra in number systems (corrected def. of $\mathbb{Q}$; added exercise 7)

Symbol	Name	Definition	Examples
$\mathbb{N}$	Natural Numbers	{1,2,3,4,…}	5; 218
W	Whole Numbers	{0,1,2,3,4,…}	0; 5; 218
$\mathbb{Z}$	Integers	{…,-2,-1,0,1,2,3,4,…}	-3; 0; 5; 218
$\mathbb{Q}$	Rational Numbers	$\{a/b \mid a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0\}$	2 (= 2/1), ¾, -2/5
$\mathbb{R}$	Real Numbers	All numbers on the number line	
$\mathbb{C}$	Complex Numbers	$\{a+bi \mid a \in \mathbb{R}, b \in \mathbb{R}\}$, with the definition that $i^2 = -1$.	3+2i; 6; 0; -3i.
$\mathbb{Z}$ /(2)	Integers modulo 2 ("the 2-clock")	{0,1} with the multiplication and addition tables given in class	
$\mathbb{Z}$ /(3)	Integers modulo 2 ("the 2-clock")	{0,1,2} with the multiplication and addition tables given in class	
$\mathbb{Z}$ /(4)	Integers modulo 2 ("the 2-clock")	{0,1,2,3} with the multiplication and addition tables given in class	
$\mathbb{Z}$ [X]	Polynomials over the Integers		$3x^2-2x+7$; 5; x^5
$\mathbb{Q}$ [X]	Polynomials over the Rationals		½ x^2 - 9
$\mathbb{R}$ [X]	Polynomials over the Reals		π x3 -
$\mathbb{C}$ [X]	Polynomials over the Complex Numbers		i x^3 -3x

Remember: $\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$ and $\mathbb{Z} \subset \mathbb{Z}[X] \subset \mathbb{Q}[X] \subset \mathbb{R}[X] \subset \mathbb{C}[X]$

Exercise 1. Develop addition and multiplication tables for $\mathbb{Z}$ /(2).

+	0	1
0		
1		

×	0	1
0		
1		

Exercise 2. Develop addition and multiplication tables for $\mathbb{Z}$ /(3).

+	0	1	2
0			
1			
2			

×	0	1	2
0			
1			
2			

Exercise 3. Develop addition and multiplication tables for $\mathbb{Z}$ /(4).

+	0	1	2	3
0				
1				
2				
3				

×	0	1	2	3
0				
1				
2				
3				

Properties of some number systems.

System	$\mathbb{N}$	W	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{Z}$ /(2)	$\mathbb{Z}$ /(3)	$\mathbb{Z}$ /(4)	$\mathbb{Z}$ [X]	Any Ring	Any field
Closure (+)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Closure (x)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Closure (-)			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Closure (÷)				✓	✓	✓	✓	✓	✓			✓
Associativity (+)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Associativity (×)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Associativity (-)							✓					
Associativity (÷)							✓					
Commutativity (+)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Commutativity (×)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Additive Identity (0)		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Multiplicative Indentity(1)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Additive Inverses			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Multiplicative Inverses				✓	✓	✓	✓	✓				✓
$0 \neq 1$		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Unique Factorization	✓	✓	✓				✓	✓		✓		

Exercise 4. Is $\mathbb{Z}$ /(2) a Field? Why or why not?

Exercise 5. Is $\mathbb{Z}$ /(3) a Field? Why or why not?

Exercise 6. Is $\mathbb{Z}$ /(4) a Field? Why or why not?

Exercise 7. For which whole numbers $m>0$ is $\mathbb{Z}$ /(m) a Field?