Rules of Exponents

Math 130 *Kovitz* 2012

Definition of a Positive Integral Exponent

$$b^n = \underbrace{b \cdot b \cdot \cdots \cdot b}_{n \text{ factors}}$$

With n, the exponent, a positive integer and b, the base, is any real number.
The result, b^n, must be a positive real number.

Multiplying Powers with a Common Base

$$b^m \cdot b^n = b^{m+n}$$

Dividing Powers with a Common Base

$$\frac{b^m}{b^n} = b^{m-n}$$

If $m = n$ the result equals 1; if $m < n$, making the resultant exponent negative, the answer may be rewritten as $\frac{1}{b^{n-m}}$.

The Power of a Power

$$(b^m)^n = b^{mn}$$

The Power of a Product

$$(ab)^m = a^m \cdot b^m$$

The Power of a Quotient

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Powers Less than 2, including Negative Powers

$$b^0 = 1 \quad (\text{provided } b \neq 0), \quad b^1 = b, \quad b^{-n} = \frac{1}{b^n}$$

Fractional Powers and Radicals

$$b^{1/n} = \sqrt[n]{b} \quad (\text{this will not be defined as a real number if } n \text{ is even and } b \text{ is negative})$$

Note: $\sqrt[n]{b}$ is called the principal nth root of b; it always has the same sign as the expression inside the radical. The square roots of b are $\sqrt{b}$ and $-\sqrt{b}$.

Thus $\sqrt[n]{b^n} = |b|$ when n is an even power; and $\sqrt[n]{b^n} = b$ when n is an odd power.

Note: $\sqrt{b^2}$ is *not* equal to b; it is equal to $|b|$.

Rational Exponents

$$b^{m/n} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m, \quad \text{provided both roots are defined.}$$

"Pull Down" Rule

$$b^r = b^w \quad \text{implies} \quad r = w, \quad \text{provided } b \text{ is a positive number not equal to 1.}$$