Solving Quadratic Equations

Math 130 *Kovitz* 2012

Definition: *A* **quadratic equation** *is an equation that can be written in the form* $\boldsymbol{ax^2 + bx + c = 0}$*, where* a*,* b*, and* c *are real numbers and* $a \neq 0$*.*

The Zero Product Property

For any real numbers a and b:

If $ab = 0$, then $a = 0$ or $b = 0$.

Solving Quadratic Equations by Factoring

Steps for solving a quadratic equation by factoring

1. Move all terms to one side of the equation (so that the other side of the equation is 0).
2. Factor completely.
3. Set each factor equal to 0 and solve each of these equations.

Solving Quadratic Equations by Completing the Square

Steps for solving a quadratic equation by completing the square.

- Clear out all fractions and all parentheses.
- Move the constant term to one side of the equation and all other terms to the other side.
- Divide both sides of the equation by the coefficient of x^2 (if it is a number other than 1).
- Take the coefficient of x, divide it by 2, square the result, and add that square to both sides of the equation.
- Factor one side of the equation into a perfect square and solve the equation. Taking the square root of both sides of the equation will introduce plus or minus in front of the number on the right side.

  This step will look like this: $(x-h)^2 = p$ gives $x-h = \pm\sqrt{p}$ and $x = h \pm \sqrt{p}$.

  When the perfect square has a plus sign after the x it will be: $(x+h)^2 = p$, giving $x + h = \pm\sqrt{p}$ and $x = -h \pm \sqrt{p}$.

The Quadratic Formula

The solutions to the equation

$$ax^2 + bx + c = 0$$

are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Which Method to Use?

Assume that a, b and c are all integers. First find the discriminant $b^2 - 4ac$.

- If the discriminant is 0, there is one rational solution obtained by writing the expression as a perfect square.
- If the discriminant is a non-zero perfect square, the equation will factor over the integers, but if the factoring is not obvious, one of the other two methods might be easier. There will be two distinct rational roots.
- If the discriminant is positive but not a perfect square, it will not factor over the rationals. Then look at $b/2a$. If $b/2a$ is an integer, completing the square will be the simplest method. Otherwise use the quadratic formula. Either way, there will be two distinct irrational roots.