Practice for Log part of the Final Exam

Math 130 *Kovitz* Spring 2014

1. Let
$$f(x) = \frac{1}{9x-2} + 5.$$
   (a) Find its inverse. Find either a formula for $f^{-1}(x)$ or a verbal string for the inverse.
   (b) Find $f(6)$, $f^{-1}(6)$, $f(f^{-1}(6))$, and $f^{-1}(f(6))$.
   (c) True or false.
   f is its own inverse, meaning that its graph is symmetric across the line $y = x$.

2. Rewrite each of the functions f, g, and h in the form $C_0 a^t$.
   $f(t) = 128\left(4^{-t}\right)$ $\quad g(t) = \left(\frac{1}{4}\right)^{t-3.5}$ $\quad h(t) = 128\left(2^{-2t}\right)$
   Then graph each of the three functions on the same axes, and plug $t = 2$ into each of the three functions and show the resulting point on the graph.

3. Estimate between two consecutive integers. $\_\_\_\_ < \log_{2.5} 12 < \_\_\_\_$

4. In each of parts (a) through (d), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 0.3562$, $\log_b 3 \approx 0.5646$, and $\log_b 5 \approx 0.8271$.
   (a) $\log_b 7.5$
   (b) $\log_b 0.4$
   (c) $\log_b 32$
   (d) $\log_b \sqrt{27b}$

5. For the previous problem, estimate b to two-decimal-place accuracy.

6. Graph $y = \log_5 x$ for $-2 \le y \le 3$, labeling the point on the graph for each integer-valued y between -2 and 3 (including -2 and 3). Also draw an arrow pointing to the asymptote. The asymptote will be portion of a certain straight line.

7. Solve for x:
$$\log_8\left(1 - \frac{x^2}{18}\right) - \log_8\left(\frac{4x}{3}\right) = -1.$$

8. The number of bacteria in a culture is increasing according to the law of exponential growth. The initial population is 125 bacteria, and the population after 43 hours is double the population after 31 hours.

   (a) How many bacteria will there be after 30 hours? After 120 hours?

   (b) Find the percent increase per hour.

   Use any of three methods to answer this:

   - Find an expression that includes a radical.
   - Use the Rule of 70 for a rough approximation.
   - Use a calculator to get a 4-decimal-place approximation.

   (c) Find the 8-hour growth factor.

   Answer with an exact radical or with an approximate decimal to 4-decimal-place accuracy.

   (d) When will there be 1000 bacteria present?

9. A population is increasing according to the law of exponential growth. The initial population is 100, and—at any time—the population is double what it was 40 minutes (2/3 hours) earlier.

   (a) How large will the population be after 4 hours? After 1 hour?

   (b) Find the percent increase per hour.

   Use either of the fo9llowing two methods to answer this:

   - Find an expression that includes a radical.
   - Use a calculator to get a 4-decimal-place approximation.

   (c) Find the 2-hour growth factor; and find the one-minute growth factor.

   Answer with an exact radical or with an approximate decimal to 4-decimal-place accuracy.

   (d) When will there be 1600 bacteria present?

10. The number of bacteria in a culture is increasing according to the law of exponential growth. The initial population is 732 bacteria, and it takes 17 minutes to double.

    (a) How many bacteria will there be after 37 minutes?

    (b) Find, to four-decimal-place accuracy, the percent increase in (i.) one minute and (ii.) one hour.

    (c) Find, to four-decimal-place accuracy, the growth factor for (i.) 8 minutes and for (ii.) one hour.

    (d) When will there be 50,000 bacteria present?

ANSWERS FOLLOW

Answers.

1. (a) $f^{-1}(x) = \dfrac{2x-9}{9x-45}$ or use the verbal string: subtract 5, take reciprocal, add 2, then divide by 9.

   (b) $5\frac{1}{52}$, $1/3$, 6, and 6.

   (c) False.

   The equations for $f(x)$ and for $f^{-1}(x)$ are not equivalent.

   Or the two verbal strings are not equivalent, since they do not give the same answer for all possible inputs.

   Or take any point a in the domain of f and check whether $f(f(a)) = a$. Try $f(f(1/9))$. The result is $f(4) = 5\frac{1}{34}$: not $1/9$.

2. All three come out to $128\left(\frac{1}{4}\right)^t$.

   The graphs are decreasing with y-intercept at $(0, 128)$ and the asymptote the positive x-axis. The point for $t = 2$ is at $(2, 8)$.

3. The inequality will be $2 < \log_{2.5} 12 < 3$ because $2.5^2 = 6.25$ and $2.5^3 = 15.625$.

4. (a) 1.0355; (b) −0.4709; (c) 1.7810; (d) 1.3469.

5. 7.00.

6. (graph not shown) The graph is in the first and fourth quadrants, decreasing, concave down, and has an x-intercept at $(1, 0)$. The asymptote is the negative y-axis.

   The requested points are $(1/25, -2)$, $(1/5, -1)$, $(1, 0)$, $(5, 1)$, $(25, 2)$, and $(125, 3)$.

7. $x = 3$.

8. (a) About 707 after 30 hours, and exactly 128,000 after 120 hours.

   (b) An expression: $100\%(\sqrt[12]{2} - 1)$; from the rule of 70: a bit less than 6%; a calculator approximation: 5.9463%.

   (c) An exact answer: $\sqrt[3]{4}$; approximately 1.5874.

   (d) After exactly 36 hours.

9. (a) Exactly 6400 after 4 hours, and approximately 283 after one hour.

   (b) An expression: $100\%\left(2\sqrt{2} - 1\right)$; a calulator approximation: 182.8246%.

   (c) For two hours: 8; for one minute: an exact answer: $\sqrt[40]{2}$; approximately a 1.01748.

   (d) After exactly 2 hours and 40 minutes, which is $2\frac{2}{3}$ hours.

10. (a) About 3.309.

    (b) (i.) About 4.1616%; (ii.) about 1054.67246%.

    (c) (i.) About 1.385674 (exactly $2^{8/17}$); (ii.) about 11.5467246 (exactly $2^{60/17}$).

    (d) After about 103.6 minutes, which is one hour, 43 minutes, and roughly 36 seconds.