Sample of Typical Final Examination Problems

Math 130 Precalculus for the May 23, 2014 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. Find a third point on the line containing the points $A(-3, 8)$ and $B(7, 2)$ that's just as far from B as A is.

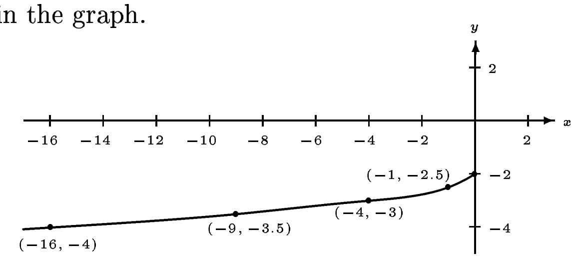
   Explain what characterizes any other point on that line, and point out which formula would be best to provide the coordinates of the unique point on that line which is located just as far from B as A is.

2. Write the standard form of the equation of a circle with endpoints of a diameter $(0, 0)$ and $(4, -6)$. State the center and the radius, and sketch the graph of the circle, labelling all intercepts with their coordinates.

3. Find an equation of the line passing through the points $\left(2, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{5}{4}\right)$. Sketch the line and label both intercepts with their coordinates.

4. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\dfrac{f(x+h) - f(x)}{h}$ and simplify the result.

5. Identify the parent function and the transformation shown in the graph. Then write an equation for the function shown in the graph.



6. For $f(x) = \sqrt{x+4}$ and $g(x) = x^2$, find $f \circ g$ and $g \circ f$.

   Find the domain of each function and the domain of each composite function.

7. Determine whether $h(x) = -5x + 3$ has an inverse function. If it does, then find the inverse function.

8. Write the quadratic function $f(x) = \frac{1}{4}x^2 - 2x - 12$ in standard form and sketch its graph.

   Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.

9. Find two positive real numbers whose product is a maximum, under the condition that the sum of the first number and three times the second number is 42.

10. Graph $y = 5^x$.

    For each of these equations, state the transformation of $y = 5^x$ that yields the new equation, and then graph the transformed equation.

    (a) $y = 5^x - 2$.

    (b) $y = 5^{x-2}$.

    (c) $y = \frac{1}{25}\left(5^x\right)$.

    Are any of these three new equations equivalent? If so, prove it algebraically and show that their graphs have the same points.

11. Which of these are equivalent? $y = \left(\frac{1}{9}\right)^{x-2}$; $y = 81\,(9^{-x})$; $y = 81\,(3^{-2x})$; $y = \frac{81}{9^x}$; $y = \left(\frac{9}{3^x}\right)^2$

12. Let $w = \log_2 a$.
    Find an expression in terms of w for:
    (a) $\log_2 a^4$
    (b) $\log_2(4a^2)$
    (c) $\log_2(4a)^2$
    (d) $\log_2(\sqrt[4]{a})$
    (e) $[\log_2 4a]^2$
    (f) $\sqrt{\log_2 a^2}$

13. (a) Simplify to a rational number:
    $$\log_2\left(16\sqrt[3]{1/4}\right).$$
    (b) Simplify to a rational number or to an exact decimal:
    $$\left[1 + 3\log_2\left(\sqrt[4]{2}\right)\right]^2.$$

14. Simplify: $16^{-2+3\log_{16}5}$. Give the answer as an exact rational number.

15. In each of parts (a) through (h), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 1.098$, $\log_b 3 \approx 1.740$, and $\log_b 5 \approx 2.5495$.

(a) $\log_b 6$	(e) $\log_b 20$
(b) $\log_b \frac{3}{5}$	(f) $\log_b(4b)^{-2}$
(c) $\log_b 125$	(g) $\log_b(5b^2)$
(d) $\log_b \sqrt{3}$	(h) $\log_b \sqrt[3]{2b}$

16. Is knowing that $\log_b 7 = 2.472$ enough to find $\log_b 10$? If so, find $\log_b 10$.

17. Solve algebraically:
    (a) $\log x + \log(x - 15) = 2$.
    (b) $\log x - \log(x - 15) = 2$.
    (c) $\log_8(x + 1) - \log_8(x - 3) = \frac{1}{3}$.
    (d) $\log 24x - \log(1 + \sqrt{x}) = 2$.

18. Decide if each statement is true or false. Then justify your answer by writing an equation.
    (a) Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.
    (b) Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.

19. The number of bacteria present in a culture is increasing according to the law of exponential growth. After 3 hours there are 100 bacteria and after 7 hours there are 400 bacteria. How many bacteria will there be after 8 hours?

20. (a) Rewrite in radian measure as a multiple of π: (i) $130°$ (ii) $-60°$

    (b) Rewrite in degree measure: (i) $\dfrac{3\pi}{2}$ (ii) $\dfrac{5\pi}{4}$ (Do not use a calculator.)

    (c) Find the length of the arc on a circle of radius 3 meters intercepted by a central angle of $150°$.

21. A carousel with a 50-foot diameter makes 4 revolutions per minute.

    (a) Find the angular speed of the carousel in radians per minute.

    (b) Find the linear speed (in feet per minute) of the platform rim of the carousel.

    (c) If an individual sat at the rim and rode for an hour, how many miles (rounded to two decimal places) would he have travelled?

22. A right triangle has an acute angle θ with $\sec\theta = \frac{3}{2}$. Find the exact values of the other five trigonometric functions of θ, in fractional form.

    Hint. First sketch a right triangle corresponding to that secant. Next use the Pythagorean Theorem to determine the third side. Then find the other five trigonometric functions of θ.

23. Simplfiy each trigonometric expression in (a)–(f) so that the answer is in terms of at most one of the six trigonmetric functions.

    (a) $\sin x \cot x$ (b) $\tan^2 x - \sec^2 x$ (c) $\csc^4 x - \cot^4 x$ (d) $\frac{1}{\cos^2 x \sin^2 x} - \csc^2 x$ (e) $\dfrac{\cos^2 x - 1}{\tan^2 x}$ (f) $\dfrac{\sin^2[(\pi/2) - x]}{\cot^2 x}$

24. Find all solutions of $\sin x - 2 = \cos x - 2$ in the interval $[0, 2\pi)$. (Get the algebraically-assisted, exact result.)

25. Find all solutions of $\sec x + \tan x = 2$ in the interval $[0, 2\pi)$.

    Answers may be presented as exact values in terms of inverse trig functions, or in degree or radian form rounded to four decimal places.

26. Find the exact (algebraically-assisted) solutions of the equation in the interval $[0, 2\pi)$. Then graph the double-angle function and the other trig function on the same axes and estimate their points of intersection. Show that these estimates are compatible with the exact solutions.

    (a) $\cos 2x - \cos x = 0$.

    (b) $\sin 2\alpha - \cos\alpha = 0$.

27. Find the exact values of the sine, cosine, and tangent of $\dfrac{11\pi}{12}$. (No credit for approximate decimal answers.)

    Some people will break the angle down as $\frac{3\pi}{4} + \frac{\pi}{6}$, while others will break the angle down as $\frac{2\pi}{3} + \frac{\pi}{4}$. Won't this make a big difference in the answers?

28. Given that $\cos u = -4/5$ with $\pi/2 < u < \pi$, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

29. What can you say about the trig functions of double the angle $323.13°$ as compared with the trig functions of double the angle $143.13°$?

30. Use the Law of Cosines to solve the triangle with sides of lengths 6 and 7, and an included angle of $120°$ between them. (Round angles to three decimal places.)