Functions of two variables. Class ________ Name _________________________ Date _____

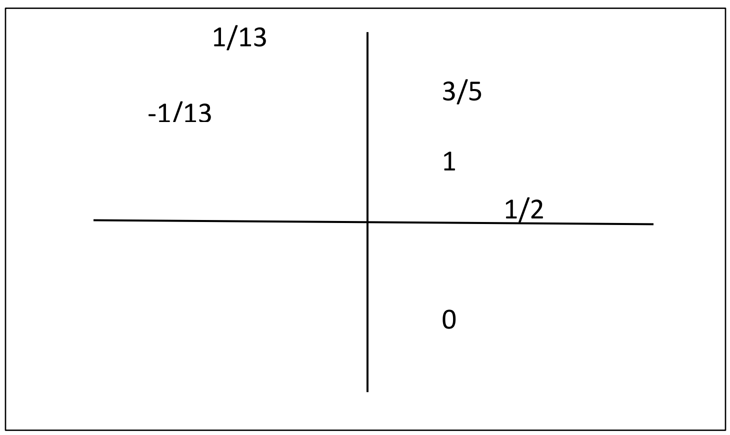
A function of two variables f(x,y) is a rule, which, given values for x and y, yields a unique value f(x,y). The domain of such a function is the set of pairs (x,y) for which the function value exists. The range of such a function is the set of all possible values of f(x,y).

Examples:

E1. $f(x,y) = xy$. The multiplication function.
E2. $f(x,y) = x+y$. The addition function.
E3. $f(x,y) = x-y$. The subtraction function
E4. $f(x,y) = x^2+y^2$.
E5. $f(x,y) = (x+y)/(x^2+y^2)$. Domain = $\{(x,y) \mid \text{either } x \neq 0 \text{ or } y \neq 0\}$
E6. $F(x,y) = x/y$. The division function. Domain = $\{(x,y) \mid y \neq 0\}$
“Graphing” functions of two variables would require a 3-dimensional graph, which we don’t have. However, we can get a picture of the function if we place function values on the x,y plane.

Here is the “graph” of the function (E5) above:

x	y	f(x,y)
1	1	1
1	2	3/5
0	y	1/y
1	-1	0
-2	3	1/13
0	0	undefined
-3	2	-1/13



An important example of a function of two variables is the <u>DIFFERENCE QUOTIENT</u> of a function f(x). The D.Q. of f(x) is defined to be a function $g(x,h) = [\ f(x+h) - f(x)\ ]\ /\ h$. The domain of this function is $\{(x,h) \mid f(x) \text{ is defined, and } f(x+h) \text{ is defined, and } h \neq 0\}$.

The DIFFERENCE QUOTIENT will be discussed on a subsequent handout.

Exercises:

	EVALUATE at (0,0), (4,3),(2,-1) and (-2,-2)
1	$f(x,y) = x-y$
2	$g(x,y) = x^2 +2xy + y^2$
3	$h(x,y) = x^2 - y^2$
4	$r(x,y) = \text{sqrt}\ (x^2+y^2)$

	GRAPH:
5	E1, above
6	E4, above
7	E5, above
8	E6, above.