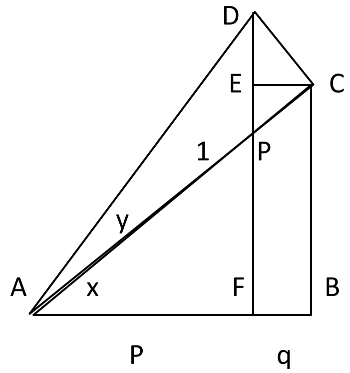
Proof of the cosine of a sum formula.



To prove: cos(x+y) = cos(x) cos(y) – sin(x) sin(y).

	Statement	Reason
1	Construct ΔACB with angle A = x, angle B = 90°, AC = 1	
2	BC = sin(x)	Def. of sin
3	AB = cos(x)	Def. of cos
4	Construct ΔACD, with D on the opposite side of segment AC from B, so that angle DAC = y, and angle ACD = 90°.	
5	Angle BAD = (x+y)	By construction, angle addition theorem.
6	From D, drop a perpendicular to AB, meeting AB in point F.	From a pt not on a line, there exists a perpendicular to that line.
7	Let AF = p. Let FB = q. Let DC=r.	
8	In ΔACD, cos(y) = AC/AD.	
9	Since AC=1, AD = 1/cos(y)	#1, #8, substitution
10	In ΔACD, sin(y) = DC/AD.	Def. of sin
11	r = CD = AD sin(y)	#10
12	r = CD = (1/cos(y)) sin (y) = sin(y)/cos(y)	#11, #9, substitution
13	In ΔADF, cos(x+y) = AF/AD	Def of cos; #4;
14	AF = p = AD cos(x+y)	#13; multiply by AD
15	AF = p = (1/cos(y)) cos(x+y) = cos(x+y) / cos(y)	#14;#9; substitution
16	Let P be the point where segment AC intersects segment DF	
17	Angle APF is congruent to angle CPD	Vertical angles are congruent
18	Angle APF is complementary to angle x	
19	Angle CPD is complementary to angle PDC	
20	Angle CDF = angle x	Complements of the same angle are congruent; #17,#18,#19
21	Construct CE perpendicular to DF at E	From a pt not on a line, there exists a perpendicular to that line.
22	In ΔDCE, sin(x) = CE/CD	Def. of sin.
23	BF = CE = q	Opposite sides of rectangle FBCE
24	q = CE = CD sin(x) = (sin(y)/cos(y)) sin(x)	#12,#22, #23
25	p + q = AF + FB = AB	Segment AB
26	cos(x+y)/cos(y) + sin(x) sin(y)/cos(y) = cos(x)	#3, #15, #24, #25, substitution
27	cos(x+y) + sin(x)sin(y) = cos(x) cos(y)	#26, multiply by cos(y)
28	cos(x+y) = cos(x) cos(y) – sin(x) sin(y)	#27, subtraction

Which was to be proved.