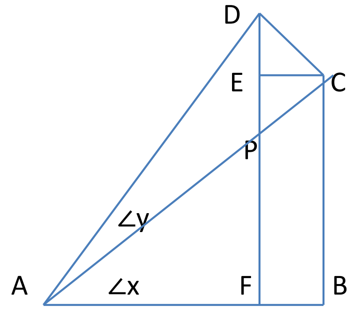
Proof of the cosine of a sum formula

To prove: cos(x+y) = cos(x) cos(y) – sin(x) sin(y)



ΔACB is a right triangle. ∠CAB = x. ∠B = 90^o. AB = 1

ΔACD is a right triangle, with ∠DAC =y, and ACD=90^o.

Segment DF is perpendicular to AB.

Segment EC is perpendicular to CB.

Figure FECB is a rectangle.

P is the intersection of DF and AC.

	Statement	Reason
1	Construct ΔACB with ∠CAB = x, ∠B = 90^o, AB= 1	Angle construction postulate.
2	AB / AC = cos(x)	cos = adjacent/hypotenuse
3	AC = 1/cos(x)	AB=1 (#1), and #2
4	Construct ΔACD, with D on the opposite side of segment AC from B, with ∠DAC =y, and ACD=90^o.	Angle construction postulate.
5	∠ BAD = ∠ (x+y)	By construction, and the Angle Addition Theorem (segment AC is interior to ∠ BAD); #4.
6	From D, drop a perpendicular to A B, meeting AB in point F	From a point not on a line, there exists a perpendicular to the line.
7	AC/AD = cos(y)	cos = adjacent/hypotenuse
8	(1/cos(x)) / AD = cos(y)	Substitution, #3, #7
9	AD = 1/ [cos(x) cos(y)]	#8, multiply both sides by AD/cos(y)
10	AF/AD = cos(x+y)	cos = adjacent/hypotenuse
11	AF = AD cos(x+y) = cos(x+y) / [cos(x) cos(y)]	#9, #10
12	Segment AC and segment DF intersect in a point P	Since ∠B is a right angle, ∠x is acute, and AC and DF are not parallel.
13	∠ APF = ∠ DPC	Vertical angles are congruent
14	∠ FDC is complementary to ∠ DPC	#4, #12
15	∠ APF is complementary to ∠ x	#1, #6
16	∠ FDC = ∠ BAC = x	Complements of the same ∠ are congruent, #14, #15
17	Construct CE perpendicular to DF from the point C	From a point not on a line, there exists a perpendicular to the line.
18	Figure FECB is a rectangle	#1,#4,#17, so it has 3 right angles
19	CE = FB	Opposite sides of a rectangle
20	DC = sin(y) AD = sin(y) / [cos(x) cos(y)]	sin = opposite/hypotenuse in ΔCED
21	CE =DC sin(x) = sin(x) sin(y) / [cos(x) cos(y)]	sin = opposite/hypotenuse in ΔCED
22	AF+FB = AB = 1	Segment addition postulate
23	cos(x+y) / [cos(x) cos(y)] + sin(x)sin(y)/[cos(x)cos(y)] = 1	#22, substitution, #11,#19,#21,
24	cos(x+y) + sin(x)sin(y) = cos(x)cos(y)	#23, multiply by cos(x)cos(y)
25	cos(x+y) = cos(x)cos(y) - sin(x)sin(y)	#24, subtraction

Which was to be proved.