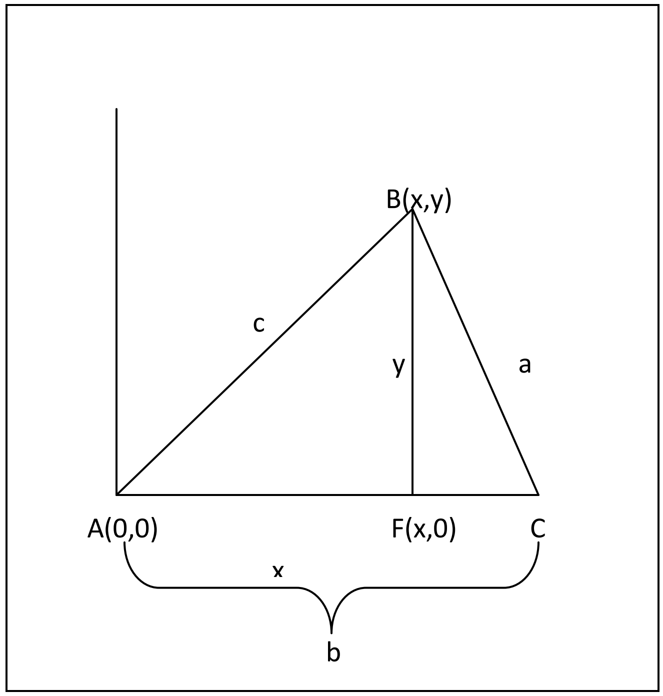
Proof of the law of cosines:
$a^2 = b^2 + c^2 - 2bc\cos(A)$



<u>Idea of proof</u>:

Use the Pythagorean Theorem in triangle BFC.

Use the fact that $x/c = \cos(A)$ and $y/c = \sin(A)$.

1	Given any triangle ΔABC, with sides $a = BC$, $b = AC$, $c = AB$	given
2	Drop a perpendicular BF from B to side AC, with F on side AC.	From a point not on a line, there exists exactly one perpendicular to the line (parallel postulate).
3	Let $x = AF$	
4	In ΔABF, $x/c = \cos(A)$, so $x = AF = c\cos(A)$.	ΔABF is a right Δ; def. of cosine
5	In ΔABF, $(BF)/c = \sin(A)$, so $y = BF = c\sin(A)$.	ΔABF is a right Δ; def. of sine
6	In ΔBFC, $a^2 = (BF)^2 + (FC)^2$.	#1; Pythagorean Theorem
7	$FC = AC - AF$	Segment addition
8	$FC = b - x$.	Substitution,#7, #1, #3.
9	$FC = b - c\cos A$	Substitution, #8, #4
10	$a^2 = (c\sin A)^2 + (b - c\cos A)^2$	Substitution,#6,#5,#9
11	$a^2 = c^2\sin^2(A) + b^2 - 2bc\cos(A) + c^2\cos^2(A)$	#10, algebra
12	$a^2 = c^2\sin^2(A) + c^2\cos^2(A) + b^2 - 2bc\cos(A)$	#11,Commutative law of addition
13	$a^2 = c^2[\sin^2(A) + \cos^2(A)] + b^2 - 2bc\cos(A)$	#12, distributive law
14	$a^2 = c^2 + b^2 - 2bc\cos(A)$	#13, $\sin^2 + \cos^2 = 1$, substitution [basic Pythagorean identity]

Which was to be proved.