Curve sketching example.

	Problem	Work	Answer
1	$f(x) = \frac{3(x-2)(x-5)}{5(x-2)(x-1)}$		
1a	At which value(s) of x is f(x) undefined?	f is undefined if its denominator is zero. 5(x-2)(x-1) = 0 when x=2 or x=1.	x=2 x= 1
1b	At which values of x=a, where f(a) is undefined, does f have a limit as $x \to a$?	Note that as $x \to 2$, but x ≠ 2, we can cancel the (x-2) factors in numerator and denominator. Remember that lim $x \to a$ f(x) DOES NOT DEPEND ON THE VALUE OF f at x=a. Therefore, if x ≠ 2, then $f(x) = \frac{3(x-5)}{5(x-1)}$. Using the second form of f(x), we can "plug in" the value x=2 to get: lim $x \to 2$ f(x) = 3((2)-5) / [5 ((2)-1)] = 3(-3)/5 = -9/5 However, at x=1, f(x) = 3(1-5)/[5(0)] = = nonzero/zero. Of the three forms below: • 0/0 is undefined, but f could have a limit there of +infinity, -infinity, or a number. • 0/[non-zero] is always zero. • [non-zero]/0 is infinite (+ , -, or both)	x=2. The point (2, -9/5) is called a HOLE, or a REMOVABLE DISCONTINUITY.
1c	At which values of x=a, where f(a) is undefined, does f have no limit?	x=1. f(1) = non-zero/0. see 1b.	x=1.
1d	What are the ***vertical asymptotes*** of f?	x=1. This is the equation of a vertical line.	x=1

<table>
<tr><td>1e</td><td>What are the <u>horizontal asymptotes</u> of f?</td><td>To find horizontal asymptotes, take $\lim_{x\to infinity} f(x)$..
In this problem, we can transform f(x) by dividing top and bottom by the same number, x. The new form of f(x) is:
$f(x) = \frac{3(1-5/x)}{5(1-1/x)}$.
Now as $x \to \infty$, numerator $\to$ 3, and denominator $\to$ 5.
So lim (f(x)) = 3/5. The line y=3/5 is the horizontal asymptote of f(x).</td><td>y=3/5</td></tr>
<tr><td>1f</td><td>What is the y-intercept of f(x) ?</td><td>The y-intercept is found by evaluating f(x) at x=0.
In this problem, f(0) = [(3)(-5)] / [5(-1)] = -15/(-5)=3.</td><td>y=3</td></tr>
<tr><td>1g</td><td>What are the critical points of f?</td><td>We need to find points x=a where f'(a) = 0.
Using the reduced form of f from (1b), we get (using the quotient rule for the derivative):
$f'(x) = \{ 5(x-1)(3) – 3(x-5)5 \} / \{ [5(x-1)]^2\} =$
$\{15x-15 -15x + 75 \} / \{ 25 (x-1)^2 \} = [60/25] /\{ (x-1)^2 \}$.
In this case, the derivative cannot equal zero.</td><td>No critical points.</td></tr>
<tr><td>1h</td><td>In which region(s) is f(x) concave upwards?</td><td>Functions are concave upwards when f''<0.
$f'(x) = (12/5) (x-1)^{-2}$.
So $f''(x) = (12/5)(-2)(x-1)^{-3} = (-24/5) (x-1)^{-3}$.
If x>1, $(x-1)^{-3}$ is positive and f'' is negative.
If x<1, $(x-1)^{-3}$ is negative and f'' is positive.</td><td>f is concave up, when x>1 and x ≠ 2.
f is concave down when x<1.</td></tr>
<tr><td>1j</td><td>Make a table of values for f(x). show x,f,f'. Use interesting points.
$f(x) = \frac{3(x-5)}{5(x-1)}$.</td><td><table>
<tr><td>x</td><td>f(x) = 3(x-5) / 5(x-1).</td><td>f'(x) = $[60/25] /\{ (x-1)^2 \}$.</td></tr>
<tr><td>3</td><td>-3/5</td><td>3/5</td></tr>
<tr><td>-1</td><td>-9/5</td><td>3/5</td></tr>
<tr><td>4</td><td>-1/5</td><td>20/75</td></tr>
<tr><td>-2</td><td>-21/(-15) = 7/5</td><td>20/75</td></tr>
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<tr><td>1k</td><td>Using everything you know, accurately sketch the graph of f(x).</td><td></td><td></td></tr>
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