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| State the following definitions, theorems, or properties. 5 points each. 10 points free. Total points this quiz: 100 |
|  | Question | Answer |
| 1 | Completeness property of the real numbers | ∀ S ⊂ **R** , if S is non-empty, and if S has an upper bound b, then S has a least upper bound b. (That is, every non-empty subset of the Real Numbers that has an upper bound, has a least upper bound.) |
| 2 | Extreme Value Theorem | If f is continuous on [a,b], then f attains its maximum on [a,b]. (also, f attains its minimum on [a,b] ). |
| 3 | Intermediate Value Theorem | If f is continuous on [a,b], and f(a) < 0, and f(b) > 0, then ∃ x ∈ (a,b) with f(x) = 0. |
| 4 | L = the limit of f(x) as x approaches the point x=a. | ∀ ε >0 ∃ δ > 0, such that ∀ x, 0 < | x-a| < δ , then |f(x)-f(z)|< ε . |
| 5 | The function f(x) is continuous at the point x=a. | (1) f(a) exists; (2) lim f(x) exists; and (3) f(a) = lim f(x) x🡪a x🡪a  |
| 6 | The derivative f’(x) is defined to be: | lim f(x+ δ) – f(x) Δ🡪0 δ |
| 7 | The function f(x), defined on the interval [a,b], has a global maximum at x=c. | ∀ x ∈ [a,b], f(x) ≤ f(c). |
| 8 | The function f(x) has a local maximum at x=c. | ∃ δ > 0, such that ∀ x ∈ (x - δ, x + δ), f(x) ≤ f(c).OR: there is a neighborhood of c, within which ∀ x, f(x) ≤ f(c). |
| 9 | The second derivative test for a local maximum at the point x=c. | If f’(c) = 0, and f’’(c) < 0, then f has a local maximum at x=c. |
| 10 | The value x=a is a critical value of the function f. | Either f’(a) = 0, or f’ is undefined at x=a. |
| 11 | The point (a,f(a)) is a critical point of the function f. | Either f’(a) = 0, or f’ is undefined at x=a. |
| 12 | The point (c,f(c)) is a “saddle point” of the function f. | f’(c) = 0, and (c,f(c) ) is neither a local minimum nor a local maximum of f. |
| 13 | The point (c,f(c) ) is an inflection point of the function f. | The second derivative f’’ changes sign at x=c. |
| 14 | Fermat’s theorem | If f is differentiable at x=a, and f has a local minimum or maximum at x=a, then f’(a) =0. |
| 15 | Rolle’s Theorem | If f is continuous on [a,b] and differentiable on (a,b), and if f(a) = f(b), then ∃ a point c ∈ (a,b) with f’(c) = 0. |
| 16 | Mean Value Theorem | If f is continuous on [a,b] and differentiable on (a,b), then ∃ a point c ∈ (a,b) with f’(c) = [f(b) – f(a) ] / (b – a). |
| 17 | Fundamental Theorem of Calculus, part #1. | Suppose f is a continuous function on [a,b], and suppose x ∈ [a,b]. Let g = . Then g’(x) = f(x). |
| 18 | Fundamental Theorem of Calculus, part #2. | If f is continuous on [a,b], and if F is any anti-derivative of f,then = F(b) – F(a). |