Elementary Set Theory and Logic. Class/Section _______ Name ______________________

1. A **set** is a collection of objects. Usually in mathematics we deal with sets of numbers of various types, or sets of points on a line, or sets of points in the plane. Things contained in a set are called **elements** of that set.
2. **Set Notation**: The braces "{" and "}" are used to signify sets.
3. **Elements**: The symbol "∈" is used to show membership in a set;
   that is, "x∈S" means "x is an element of the set S".
   Examples: Obama ∈ {presidents of the USA}. 3 ∈ ℕ (here, ℕ = the set of Natural Numbers}.
4. **Equality**: Two sets are equal iff the two sets have exactly the same elements.
5. A *set* may be **defined** in at least three ways:
   a. By l**isting** the elements: S = {2,3,17}
   b. By showing the elements as some sort of easily identifiable **pattern**: E = the set of even natural numbers = {2,4,6,8,…}
   c. By showing the elements as **elements of a bigger so-called "universe" set**, with a **rule**:
      Example: The set E of even natural numbers = {x ∈ ℕ | for some y ∈ ℕ, x = 2y}.
      Example: The graph G of a line "y = 3x -2" is the set of points
      { (x,y) | x∈R, y∈R, and y=3x-2 }.

   The symbol "|" , sometimes written ":", is read "**such that**".
   The first example above may be read, in words, as:
      "The set E of even natural numbers is the set of all x which are elements of N, such that, for some y which is an element of N, x is equal to 2 times y".
6. **Set inclusion (subsets):** If A and B are sets, and if every element of A is also an element of B, then we say that A is a subset of B. This is written: A ⊆ B or A ⊂ B. Both symbols are used. Some authors use "A ⊂ B" to mean "A is a subset of B, but there is at least one element of B that is not an element of A". We won't do that in this course. That case is read "A is a proper subset of B".
   Example: the set of Natural Numbers is a subset of the set of Rational Numbers: ℕ ⊂ ℚ.
   This means, that if a number is a natural number, then it is also a rational number.
   So, 3 ∈ ℕ. Therefore, 3 is a rational number (because we can write 3 = 3/1 and 3/1 ∈ ℚ.
7. Set **intersection** and Set **union**. If A and B are sets, we can form two new sets as follows:
   The **intersection** of A and B, written A ∩ B, is the set of elements that are in BOTH A and B.
   That is, A ∩ B = {x | x ∈ A, and also x ∈ B}.
   The **union** of A and B, written A ∪ B, is the set of elements that are in EITHER in A or B, or both. That is, A ∪ B = {x | x ∈ A, or x ∈ B}.

LOGIC:

A few symbols from logic follow. Suppose P,Q, and R are statements. And suppose P(x) is a statement about P. Example: P(x) = "x is a natural number which is a multiple of 3".
"If P is true, then Q is true" is written P ⇒ Q.
"P is false" is written "~P".
"P is true, and Q is true" is written "P AND Q"
"Either P is true, or Q is true, or both are true" is written "P OR Q"
"For every x in the set A" is written "∀*x* ∈ A"
"There exists x in the set A" is written "∃ x ∈ A"
Some laws of logic:
~ (P AND Q) = ~P OR ~Q
~ (P OR Q) = ~P AND ~Q
~ (∀x, P(x)) = "∃ x , ~ P(x)