MATH 140 Spring 2015 Final exam practice problems

1. Find the following limits, or show that they do not exist:

   (a) $\displaystyle\lim_{y\to 2} \frac{y^2-4}{y^2-y-2}$

   (b) $\displaystyle\lim_{x\to 0^+} \frac{x}{\sqrt{1+x}-1}$

   (c) $\displaystyle\lim_{x\to 1^-} \frac{x}{\ln x}$

   (d) $\displaystyle\lim_{h\to 0} \frac{e^{x+h}-e^x}{h}$

   (e) $\displaystyle\lim_{x\to -\infty} \frac{5x^3+x-1}{2x^3-7}$

   (f) $\displaystyle\lim_{z\to 1^+} \frac{z-3}{z-1}$

   (g) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{9x^3-2}}{x^2+4}$

   (h) $\displaystyle\lim_{h\to 0} \frac{2(x+h)^3-2x^3}{h}$

2. Find the derivative of the following functions.

   (a) $e^{\sqrt{x^2+1}}$

   (b) $\ln(x^2e^x)$

   (c) $\sin^3(2\theta)$

   (d) $x\left(x+\dfrac{\tan x}{x}\right)$

   (e) $f(x) = \displaystyle\int_x^1 (2+t^4)^5\, dt$

   (f) $g(x) = \displaystyle\int_5^{x^3} \cos^2 t\, dt$

   (g) Find dy/dx if $2x^2+x+xy=1$.

   (h) Find dy/dx if $x\sin y + y\sin x = 4$.

3. Evaluate the following integrals.

   (a) $\displaystyle\int \frac{x^4+3x}{x^2}\, dx$

   (b) $\displaystyle\int \frac{\cos x}{\sin x}\, dx$

   (c) $\displaystyle\int (1+e^x)(1-e^x)\, dx$

   (d) $\displaystyle\int \frac{\sqrt{\ln x}}{x}\, dx$

4. Find $f(x)$ if $f'(x) = 3 + \frac{1}{x}$ and $f(1) = 2$.
5. Find $g(x)$ if $g'(x) = x - \sin x$ and $g(0) = 7$.
6. Let $f(x) = x^4 - 8x^2 + 8$.
   (a) Find the intervals of increase and decrease.
   (b) Find the intervals where f is concave up, and where it is concave down.
   (c) Sketch a graph that fits your information.
7. Let $f(x) = x^5 - 15x^3$.
   (a) Find the intervals of increase and decrease.
   (b) Find the intervals where f is concave up, and where it is concave down.
   (c) Sketch a graph that fits your information.
8. Let $f(x) = \frac{x+2.5}{x^2-4}$.
   (a) Find the equations of any horizontal and vertical asymptotes.
   (b) Find the intervals of increase and decrease.
   (c) Sketch a graph that fits your information.
9. Let $f(x) = \frac{2x^3+1}{x^3-1}$.
   (a) Find the equations of any horizontal and vertical asymptotes.
   (b) Find the intervals of increase and decrease.
   (c) Sketch a graph that fits your information.
10. Use a linear approximation to estimate $(1.001)^9$. Be clear about which function you are using ($f(x)$) and the point where you are taking the linearization (the a in the textbook).
11. Use a linear approximation to estimate $e^{0.2}$. Be clear about which function you are using ($f(x)$) and the point where you are taking the linearization (the a in the textbook).
12. Approximate $\displaystyle\int_0^4 \sqrt{8x+1}$ using a Riemann sum with 4 terms ($n = 4$) and taking the sample point x_i^* in each subinterval to be the left end point of that subinterval. (That is, estimate the integral using 4 rectangles and left endpoints.) (**No credit** for solving the integral exactly using the Fundamental Theorem; we want an approximation.)
13. Approximate $\displaystyle\int_0^6 \frac{1}{x^2+1}$ using a Riemann sum with 3 terms ($n = 3$) and taking the sample point x_i^* in each subinterval to be the midpoint of that interval. (**No credit** for solving the integral exactly using the Fundamental Theorem; we want an approximation.)
14. What is the point on the hyperbola $xy - y = 4$ that is closest to the point $(1, 0)$?
15. You want to build a rectangular box with a square base out of sheet metal. You are going to use 2 pieces of sheet metal for the bottom of the box to reinforce it, and only a single piece of sheet metal for all of the sides and the top. If you want to use no more than 36 sq. ft. of material, what is the largest possible volume you can enclose?

16. Let $f(x) = x^3 + 4x - 7$.
    (a) Use the Intermediate Value Theorem to show that $f(x)$ has at least one root.
    (b) Use the Mean Value Theorem or Rolle's Theorem to show that $f(x)$ has at most one root.
17. Let $g(x) = e^x + x + 2$.
    (a) Use the Intermediate Value Theorem to show that $g(x)$ has at least one root.
    (b) Use the Mean Value Theorem or Rolle's Theorem to show that $g(x)$ has at most one root.
18. A spherical soap bubble is slowly shrinking. If its surface area is decreasing at a rate of 50 square millimeters per second, how quickly is the radius decreasing when the surface area is 1000 square millimeters?
19. A car drives along an elliptical track. The track can be modeled by the equation $x^2 + 5y^2 = 14$, where x and y are measured in kilometers of distance from the center of the track. As the car passes the point $(3, 1)$, the x-coordinate is increasing at a rate of 1.5 km/min. How quickly is the y-coordinate changing at that point?
20. Find the area of the region bounded by the curves $y = 2x^2$ and $y = 4 + x^2$.
21. Find the area of the region bounded by the curves $y = e^x$, $y = x^2$, $x = 0$, and $x = 2$.
22. Consider the region bounded by the curves $y = 1 - x^2$ and $y = 0$. What is the volume of the solid obtained by rotating this region about the line $y = 0$?
23. Consider the region bounded by the curves $y = x^2$ and $x = y^2$. What is the volume of the solid obtained by rotating this region about the line $y = 0$?