UMass Boston | Exam #1
Math 260, Summer 2012 | August 6, 2012

Name: .... Student ID: ....

Directions:

(1) PRINT your name on the blue book and on this test on the designated spot.

(2) This test is closed-book, no-calculator (don't panic, you won't need one!)

(3) Partial credit may be given for an incomplete answer, but NO CREDIT is given for an answer that: is not relevant to the question asked, OR does not give an explanation when this is asked for, OR does not show the computational work, OR cannot be read or understood. (Be wise: keep the grader happy!)

(4) Explanations should be *succinct*: "brief and to the point."

(5) This test lasts 90 minutes and is worth 100 points, with the point distribution for each problem denoted by ( )'s throughout the test.

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(1) **(12 points)** Define the following:
   (a) Linear subspace of $\mathbb{R}^m$.
   (b) Basis of a linear subspace.
   (c) Coordinates of a vector in a basis.
   (d) Linear transformation between linear subspaces.

(2) **(16 points)** Find a polynomial of degree two $(a + bx + cx^2)$ whose graph passes through the points $(1, -1)$, $(2, 3)$, and $(3, 13)$.

(3) **(30 points)** Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ -2 & 0 & 4 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

   (a) Find the reduced row-echelon form of A.
   (b) Find an invertible matrix U such that $UA = \mathrm{rref}(A)$.
   (c) Find a basis of the null space of A.
   (d) Find a basis of the column space of A.
   (e) For which vectors b is the system $AX = b$ consistent?

(4) (**12 points**) Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & 1 \end{bmatrix}$$

Compute
   (a) A^{-1}
   (b) $ABA^{-1} + ACA^{-1}$

(5) (**30 points**) Let

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \qquad \text{and} \qquad w = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

be vectors in $\mathbb{R}^3$.
   (a) Show that $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis of $\mathbb{R}^3$.
   (b) Find the coordinates of w in the basis $\mathcal{B}$.
   (c) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation whose matrix in the basis $\mathcal{B}$ is given by

$$[T]_{\mathcal{B},\mathcal{B}} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

   Find the matrix of T in the canonical (standard) basis of $\mathbb{R}^3$.
   (d) Is the transformation T an isomorphism?